Topological Electronic Devices

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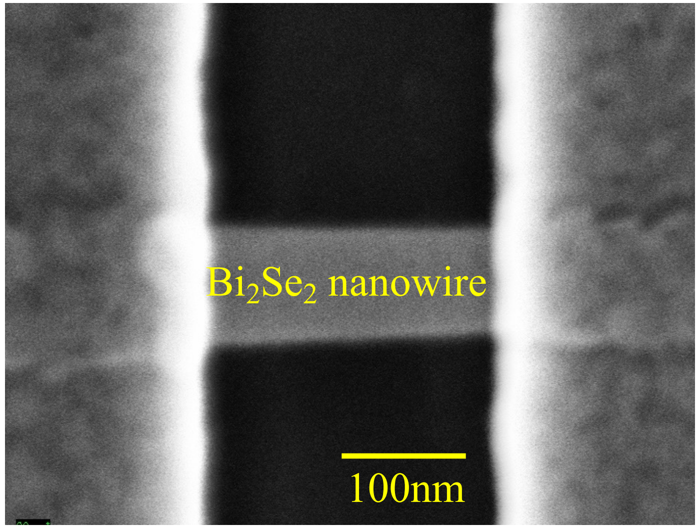

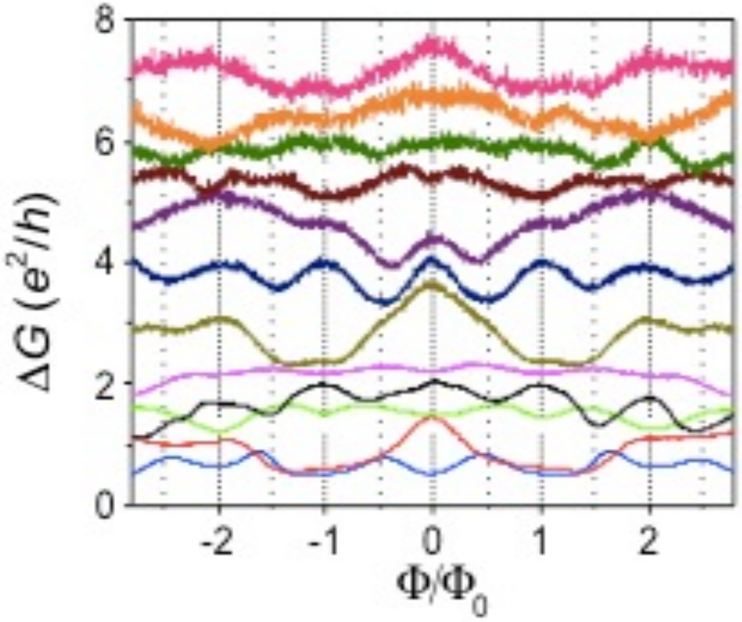

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Why Devices?

- Societal impact
- Funding
- Focuses direction of research

Focus on topological insulators

- Band gap
- Surface States
- Lack of backscattering
- Magneto-electric effect
- Spin-momentum locking
- Accessible materials

→ Great for studying charge & spin transport devices

What Devices?

"Electronic devices are components for controlling the flow of electrical currents for the purpose of information processing and system control."

- FETs
- Interconnects
- pn junctions
- Superconducting/Qubit
- Magnetic
- Emergent/tunable quantum states

Requirement for Devices:

- Dominant surface states
- Tunability (magnetic, electronic, strain, etc)
- Good material properties: low disorder, clean interfaces, etc

Outline

- Device fabrication, tunability and achieving surface-state regime
- Interconnects, FETs
- TI Josephson Junctions
- Emergent Phenomena (finite momentum states)
- TI Magnetic Devices (Tomorrow!)

Goals:

- Understand promise & limitations of topological devices
- Clarify typical experimental signals, challenges, and opportunities for transport in topological insulators

References:

•M. Gilbert, “Topological electronics,” Comm. Phys. 4, 70 (2021)

•Goldhaber-Gordon, Rechtsman, Mason, Armitage, Future Directions Workshop series: “Topological Sciences,” (2019)

•Breunig and Ando, “Opportunities in topological insulator devices,” Nat. Reviews Physics 4, 184 (2022)

•Beach, Reig-I-Plessis, MacDougall, and Mason, “Asymmetric Fraunhofer spectra in a TI-based Josephson junction,” J. Phys. Cond-Matt., 33, 425601 (2021).

•Chen, Park, Gill, Xiao, MacDougall, Gilbert, Mason, “Finite momentum Cooper pairing in 3D topological insulator Josephson junctions,” Nat. Comm. 9, 3478 (2018).

•Cho, Dellabetta, Zhong, Schneeloch, Liu, Gu, Gilbert, and Mason, “Aharonov-Bohm Oscillations in a Quasi-Ballistic 3D TI Nanowire,” Nat. Comm., 6, 7634 (2015).

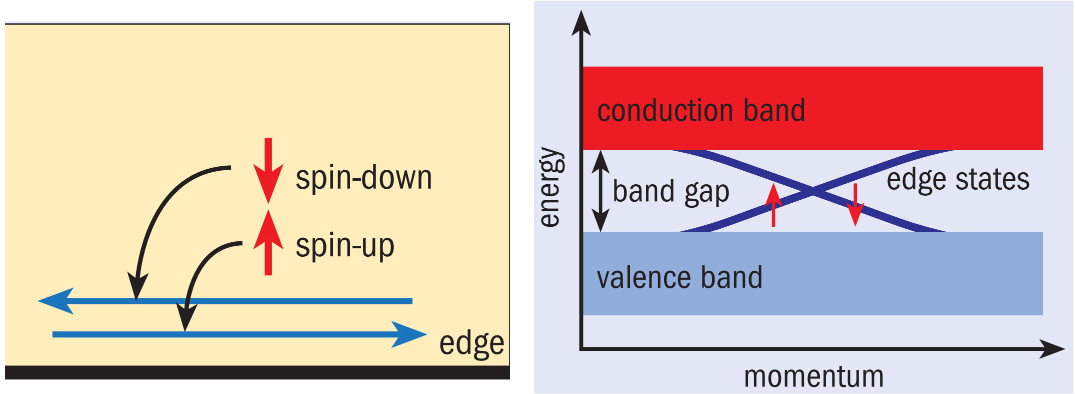
•Cho, Dellabetta, Yang, Xu, Valla, Gu, Gilbert, and Mason, “Symmetry Protected Josephson Supercurrents in Three-Dimensional TIs,” Nat. Comm., 4, 1689 (2013).

3D Topological Insulators

In 2005, it was predicted that a simple insulator with band inversion could have topological properties (even for zero applied magnetic field)

2D TIs (edge states):

e.g., HgTe

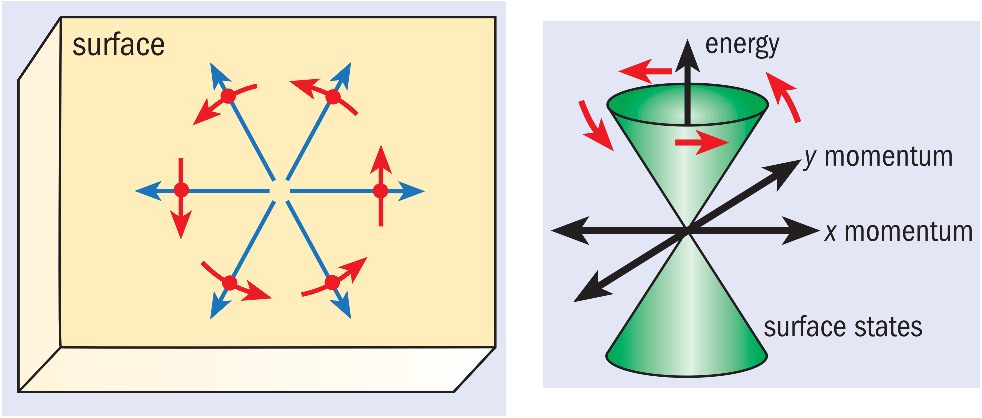


Kane, Moore, Phys. World 2011

3D TIs (surface states):

e.g., BiSe

- Strong spin-orbit coupling
- Insulating bulk
- Spin-momentum locked conducting surface states



Topological Materials

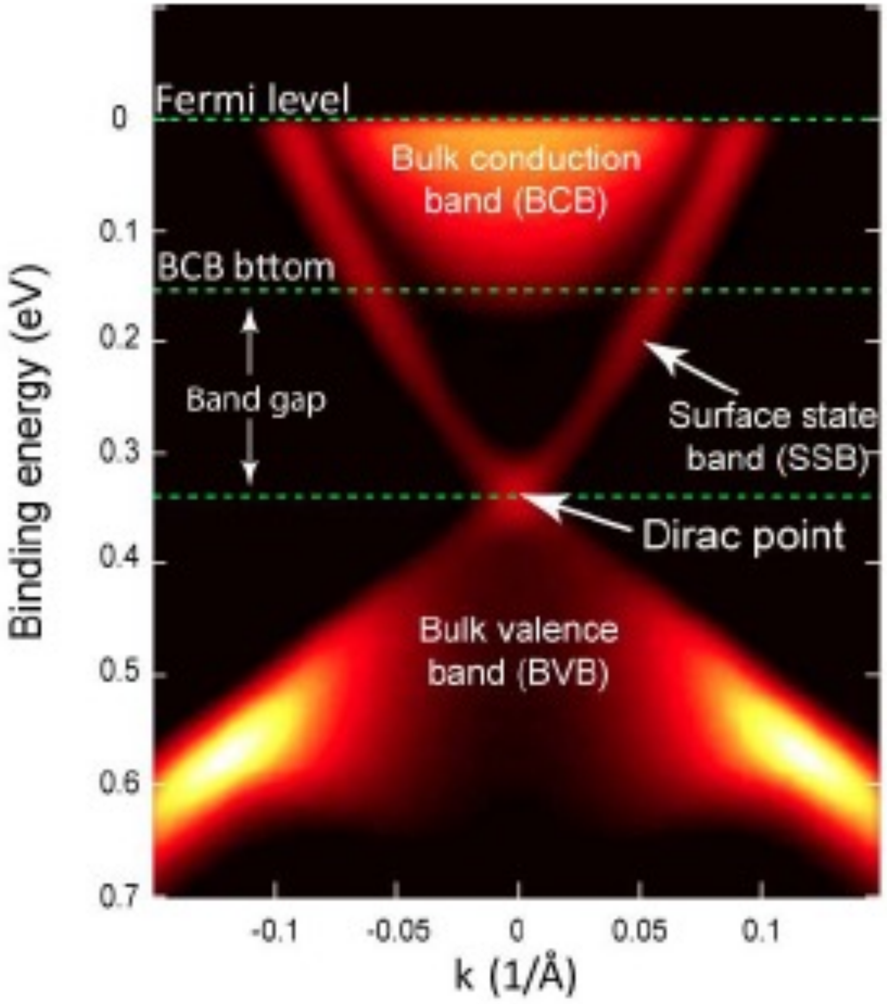
Layered Chalcogenides	3D Topological Insulator: Bi_2Se_3, Bi_2Te_3, Sb_2Te_3, Bi_2Te_2Se, $(Bi,Sb)_2Te_3$, $Bi_{2-x}Sb_xTe_{3-y}Se_y$, Sb_2Te_2Se, $TlBiSe_2$, $TlBiTe_2$, $TlBi(S,Se)_2$, $PbBi_2Te_4$, $PbSb_2Te_4$, $GeBi_2Te_4$, $PbBi_4Te_7$,	2D Topological Insulator: 1T'-WTe_2	*Magnetic 3D Topological Insulator (Axion Insulator):* Cr doped $(Bi_{1-x}Sb_x)_2Te_3$, V doped $(Bi_{1-x}Sb_x)_2Te_3$, $MnBi_2Te_4$		
Layered Chalcogenides	Weyl Semimetal: T_d-WTe_2, T_d-$MoTe_2$	Higher-Order Topological Insulator: 1T'-WTe_2, 1T'-$MoTe_2$	**Higher-Order Topological Superconductor:** $MoTe_2$, $FeTe_{1-x}Se$		
Other Chalcogenides	2D Topological Insulator: (Hg,Cd)Te Quantum Wells	Topological Crystalline Insulator: $Pb_{1-x}Sn_xTe$	*Chern Insulator:* Mn doped HgTe		
Pnictides	3D Topological Insulator: $Bi_{1-x}Sb_x$	Weyl Semimetal: TaAs, TaP, NbP, NbAs	Dirac Semimetal: Cd_3As_2, MoP		
Heusler and Half-Heuslers	*Magnetic Weyl Semimetal:* GdPtBi, NdPtBi, LuPdBi, Co_2MnX (X = Si, Ge, Sn), Co_2TiX (X = Si, Ge, Sn)		Quadratic Band Touching: YPtBi, ScPtBi		
Oxides	Dirac Semimetal: $SrIrO_3$	Quadratic Band Touching: $Pr_2Ir_2O_7$	Double Dirac Semimetal: $Sn(PbO_2)_2$, Pb_3O_4, $Mg(PbO_2)_2$, Bi_2AuO_5		
Oxides	Kramers-Weyl Semimetal: Ag_3BO_3	**Topological Superconductor:** Sr_2RuO_4	Magnetic Weyl Semimetal: $Nd_2Ir_2O_7$		
Elemental	2D Topological Insulator: monolayer hexagonal-Sn, Sb	Weyl Semimetal: Se, Te (under pressure)	Quadratic Band Touching: β-Sn	Higher-Order Topological Insulator: Bi	
Graphene and non-graphene-based moirés	2D Topological Insulator: $MoTe_2/MoTe_2^*/WTe_2$	**Superconductor in Topological Bands:** Magic angle twisted bilayer graphene	*Chern Insulator:* Magic angle twisted bilayer graphene/hBN, ABC trilayer graphene/hBN	2D Topological Insulator: Magic angle twisted bilayer graphene	
Other	2D Topological Insulator: BiH_3	Weak 3D Topological Insulator: β-Bi_4I_4	Topological Kondo Insulator: SmB_6, YbB_{12}	Dirac Semimetal: Na_3Bi	Triple Point Dirac Semimetal: WC
Other	Double Weyl Semimetal: $SrSi_2$	Double Spin-1 Weyl Semimetal: RhSi	*Magnetic Weyl Semimetal:* Mn_3Ge, Mn_3Sn	Topological Crystalline Insulator: α-Bi_4Br_4	

Table 1 Topological Materials, categorized by broad classes. 3D topological insulators (blue), 2D topological insulators (yellow), topological semimetals (orange), higher-order topological insulators (green), topological crystalline insulators (pink), and topological superconductors (purple). Magnetic materials are indicated by italics and superconductors by boldface. Inspired by a table of 2D materials in (Geim & Grigorieva, 2013).

Goldhaber-Gordon, Rechtsman, Mason, Armitage, Future Directions Workshop series: "Topological Sciences," (2019)

3D TI: Bi_2Se_3

One of first, and best, examples of 3D TI is Bi_2Se_3:



Angle resolved photoemission (ARPES) experiment on Bi_2Se_3.

•Dirac surface bandstructure and the topologically protected states evident.

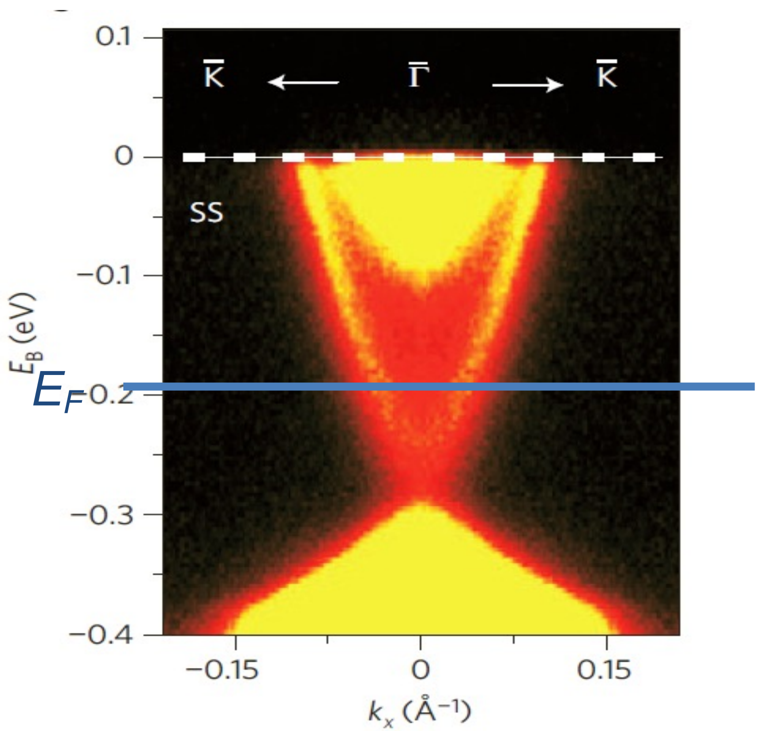
•Gap between the bulk valence band and bulk conduction band is 0.35 eV.

•Topologically protected states are evident at room temperature

•Spin-resolved ARPES → spin-momentum locked surface states

D. Hseih et al., Nature **452**, 970 (2008).
J. G. Analytis et al., PRB **81**, 205407 (2010).
H. Zhang et al., Nat. Phys. **5**, 438 (2009).

Bi_2Se_3: large bulk contribution to conductance



Xia, Hasan et al. (2009)

Fermi energy of as-grown Bi_2Se_3 is not in the gap due to Se vacancies (bulk is a metal, not an insulator)

It is difficult to measure only surface states, determine topological properties in 3D TIs via standard transport

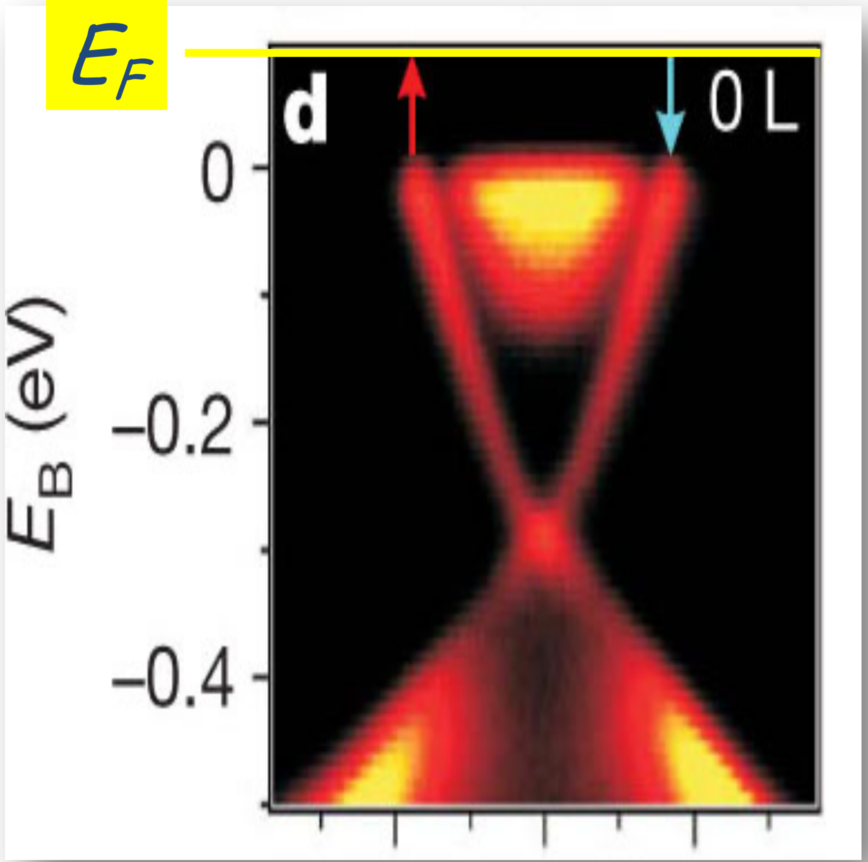
Problem common to most 3D TIs found to date

Solutions:

- Gating (top and back) [sometimes not strong enough]
- Doping with Sb, Te, e.g., BST, BSTS [may introduce disorder]
- Chemical doping [hard to control]
- Heterostructures, e.g. BiSe/BiTe [may change properties]

Or use other materials (non-chalcogenide) e.g. SmB_6 [Kondo insulator; complicated bulk behavior]

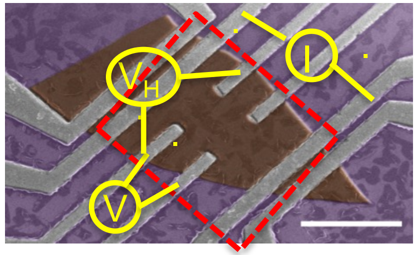
Device fabrication and approaching TI regime



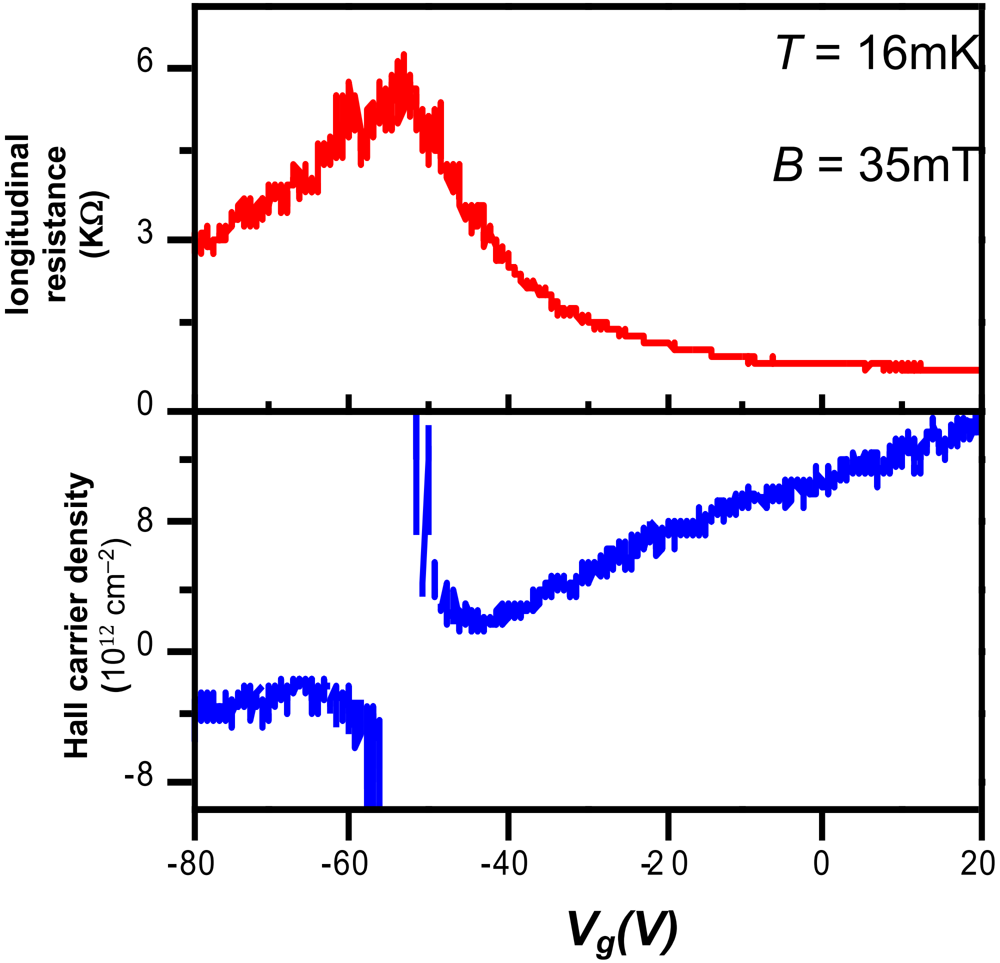
1. a thin(~10nm) **Sb**-doped Bi_2Se_3 ($Bi_{1.33}Sb_{0.67}Se_3$)
   - Sb compensates for Se vacancies (grown by Gu)
   - Exfoliate thin flakes
2. Deposit contacts
3. Chemical doping with F4TCNQ
   - strong electron affinity, so electrons transfer to interface layer
   - see Kim, Cho *et al*, Nat. Phys. **8**, 2012
4. Apply backgate through 300nm SiO_2 at low temperature

Device characterization - Hall measurement

Hall measurements to make sure that we can access the topological regime



$R_{\mathrm{Hall}} = -1/(ne)$



Device characterization - Hall measurement

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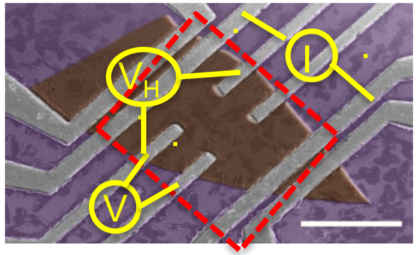

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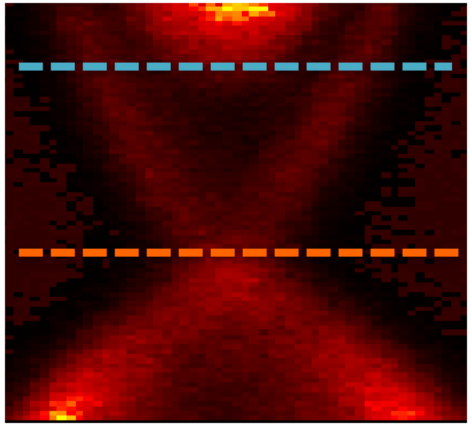
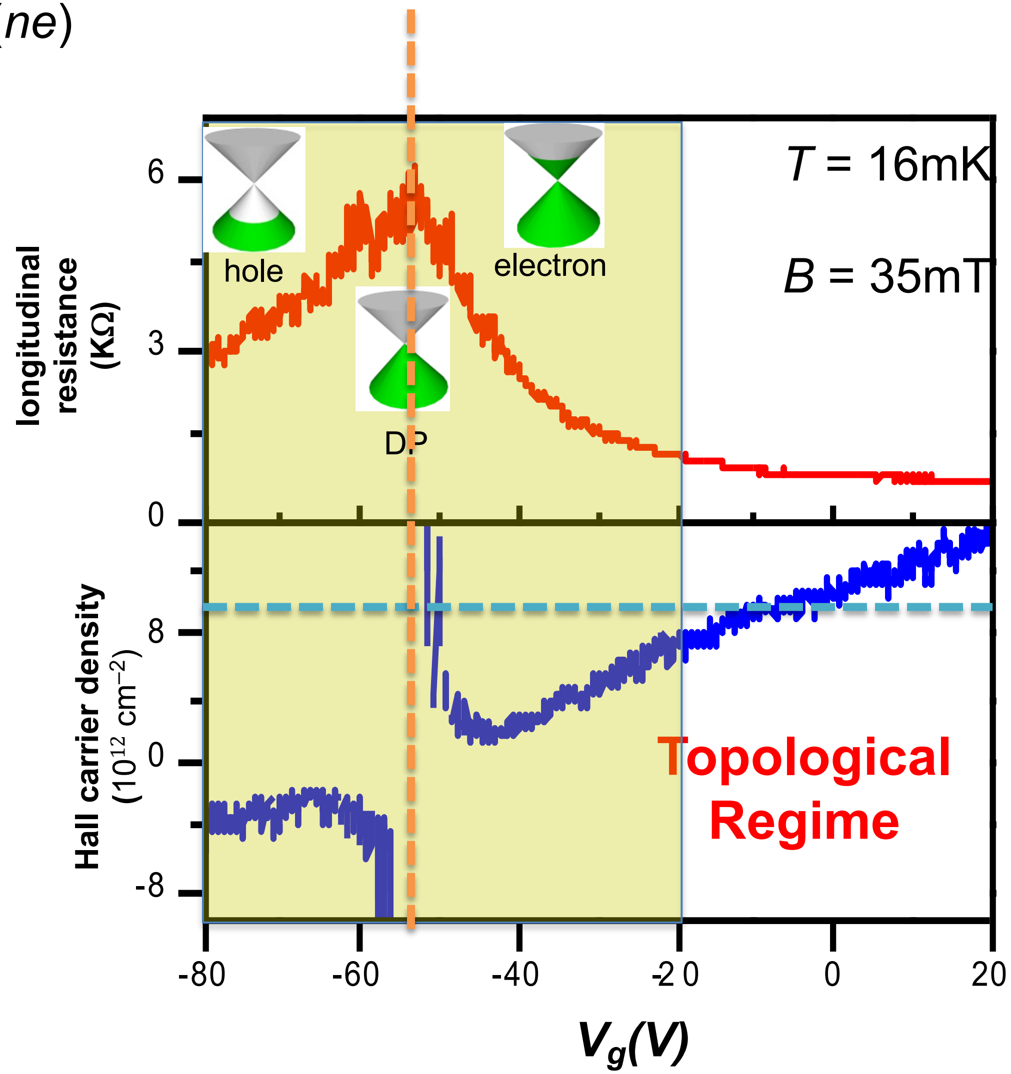
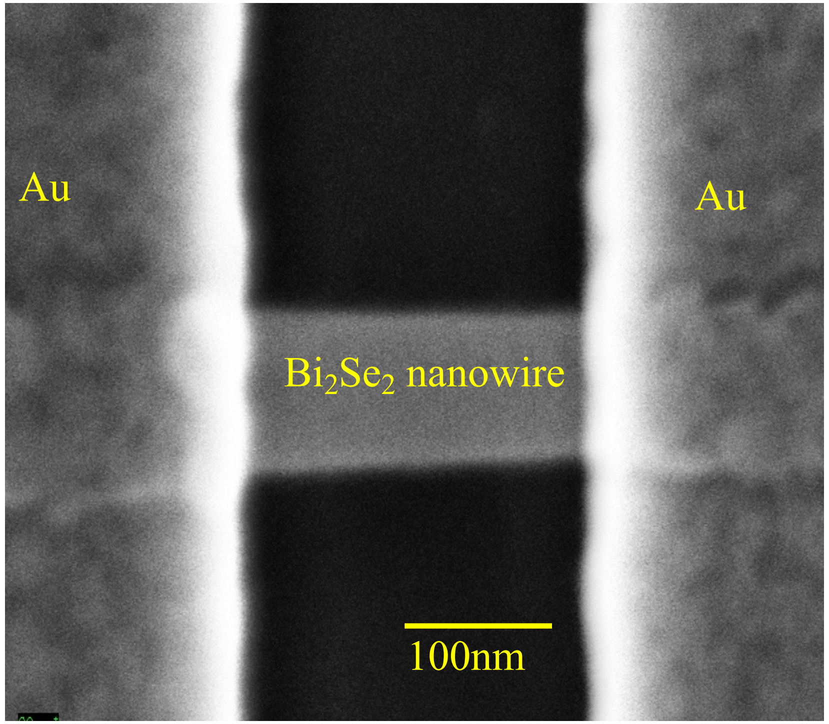




- **Dirac point** - a peak in ρ_{xx} and sign change in n_H near V_g ~ -55V as charge carrier type changes.
    - Bottom of bulk conduction band: $n \sim 0.8\text{x}10^{13}/\text{cm}^2$ near $V_g \sim -18$V.

Fabricating BiSe Devices

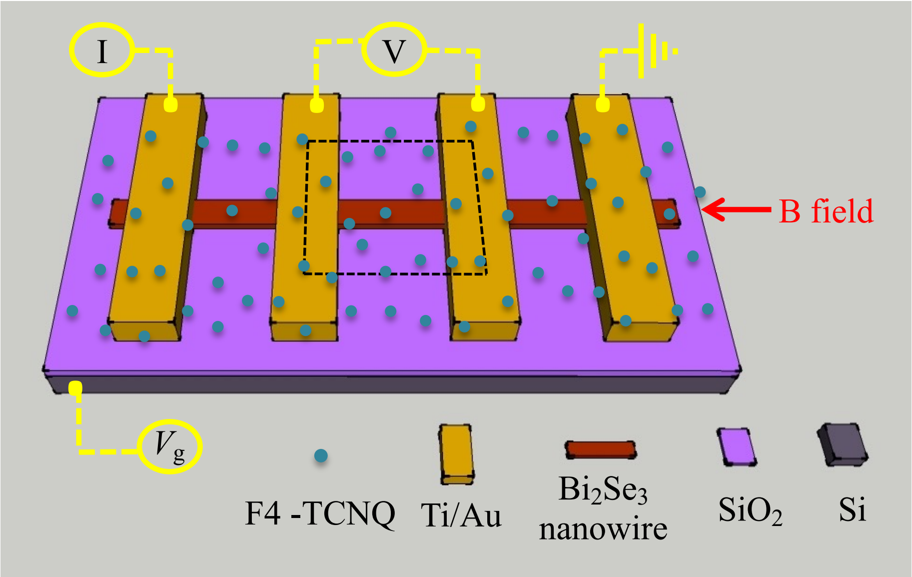


SEM image of TI (Bi_2Se_3) nanowire device

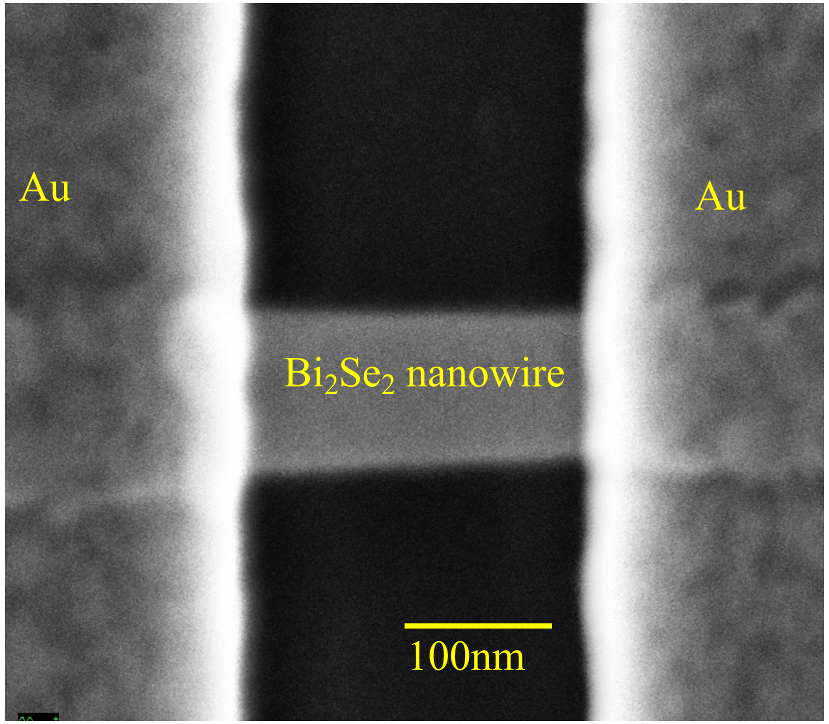
- Scotch tape exfoliate, hunt for nanowires

Length = 200nm, width = 110nm, thickness = 15nm

Molecular doping to deplete bulk states



Fabricating BiSe Devices



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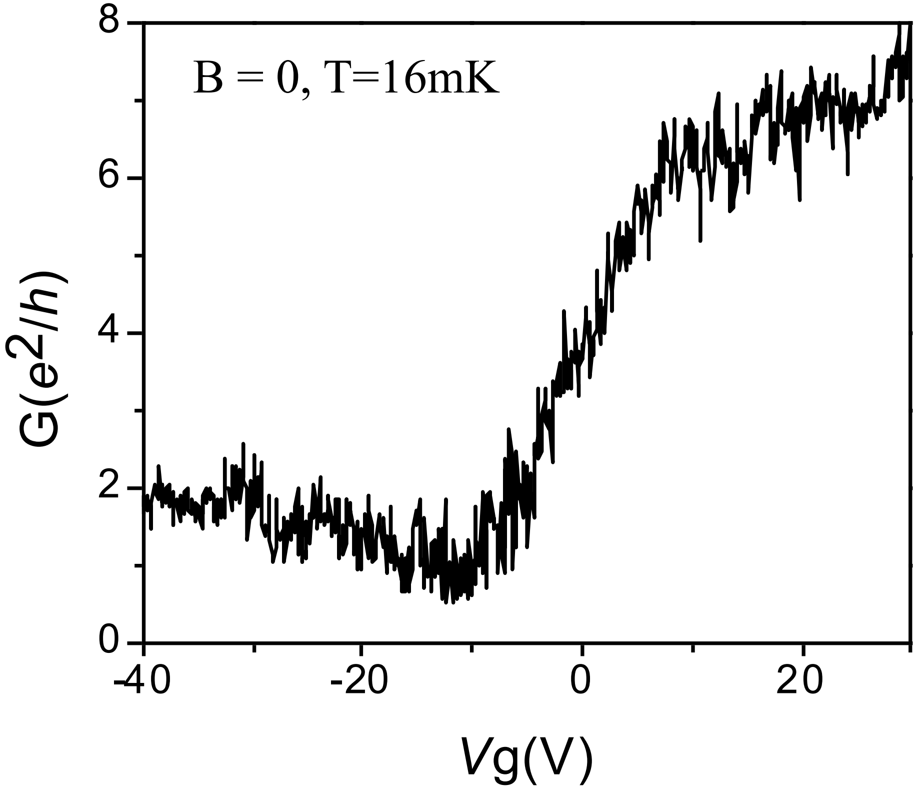


Length = 200nm, width = 110nm, thickness = 15nm

High conductance in nanowire!

What are other unique properties of the surface state?

How do we know we are measuring the surface state in transport?

3D TI Surface States

Other evidence of surface states:

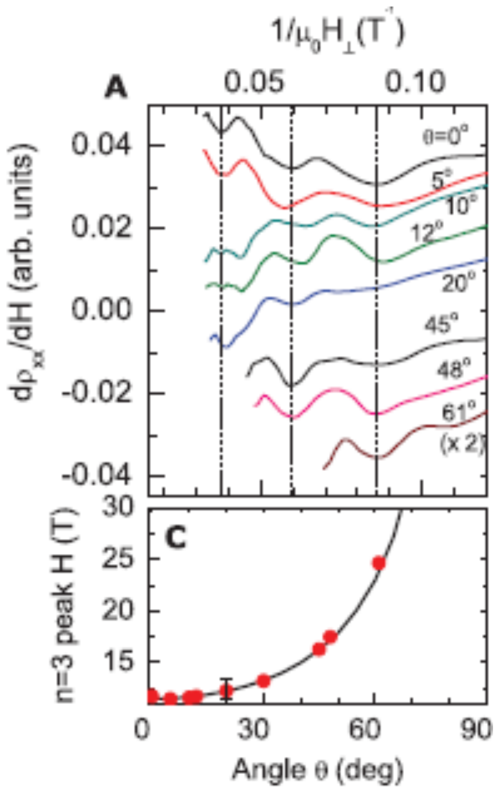
Shubnikov de Haas Oscillations (Bi_2Te_3)

Fermi surface quantization in magnetic field:

$$2\pi(n+\gamma) = S_F \frac{\hbar}{eB}$$

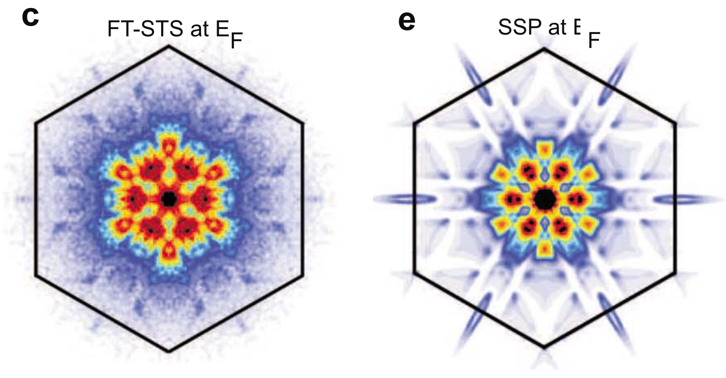
S_F = cross section of Fermi surface, g = phase factor

For 2D state, only normal component of B contributes to oscillations: B → Bcosθ



Ong, *Science* (2010)

STM: lack of backscattering (BiSb)

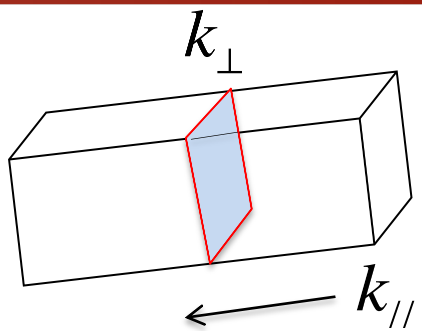


Fourier transform of scattering off defect similar to models of k-k’ suppression

Yazdani, *Nature* (2010)

Aharonov-Bohm oscillations in surface states

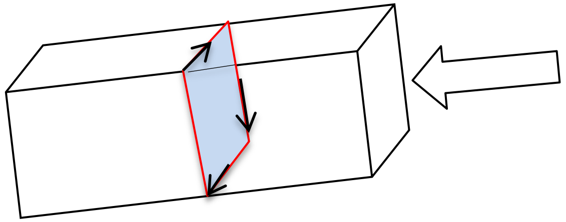
Nanowires: quantization of $k_\perp$, 1D sub-bands



$$E = \pm \hbar v_F \sqrt{k_{//}^2 + k_\perp^2} \qquad k_\perp = l\frac{2\pi}{C} \qquad l = \pm 0, 1, 2, \ldots$$

For a hollow wire, electrons in each sub-band traverse circumference of wire

Now apply magnetic field through hollow wire:



$$\Psi \rightarrow \Psi \exp\left(i \frac{\oint eB \cdot Area}{\hbar} \right) = \Psi \exp\left(i 2\pi \frac{e\Phi}{h} \right)$$

Aharonov-Bohm effect: Conductance oscillates as a function of magnetic flux enclosed by the ring, where the period of oscillation is $\Phi_0 = h/e$

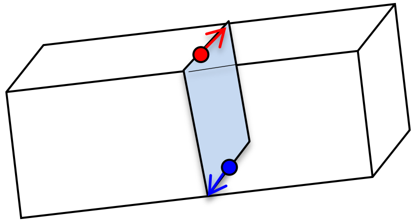
$$k_\perp = \frac{2\pi}{C}\left(l - \frac{\Phi}{\Phi_0} \right)$$

For our samples,
Area ~ 10^{-15} m^2, Φ_0 ~ 2.5T

Requires:
- Well-defined path around circumference
  → hollow wire or surface states
- Phase coherence
- Ballistic or quasi-ballistic transport

AB Oscillations: Signatures of the Gapless 1D-Topological Mode

In TIs, spin-momentum locking gives electrons an extra phase of π

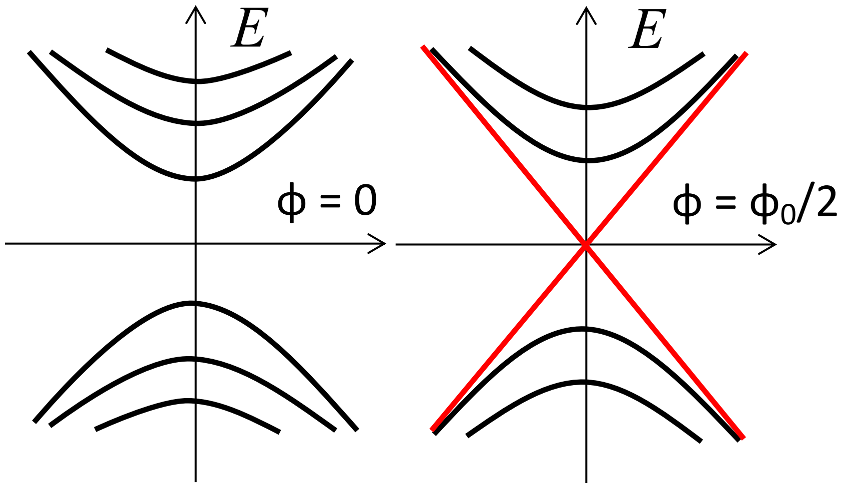


$$k_\perp = \frac{2\pi}{C}\left(l + \frac{1}{2} - \frac{\Phi}{\Phi_0}\right)$$

Leads to gap at $\Phi = 0$ for 1st subband

But extra phase cancelled by AB phase for $\phi = \phi_0/2$, leading to reappearance of mode

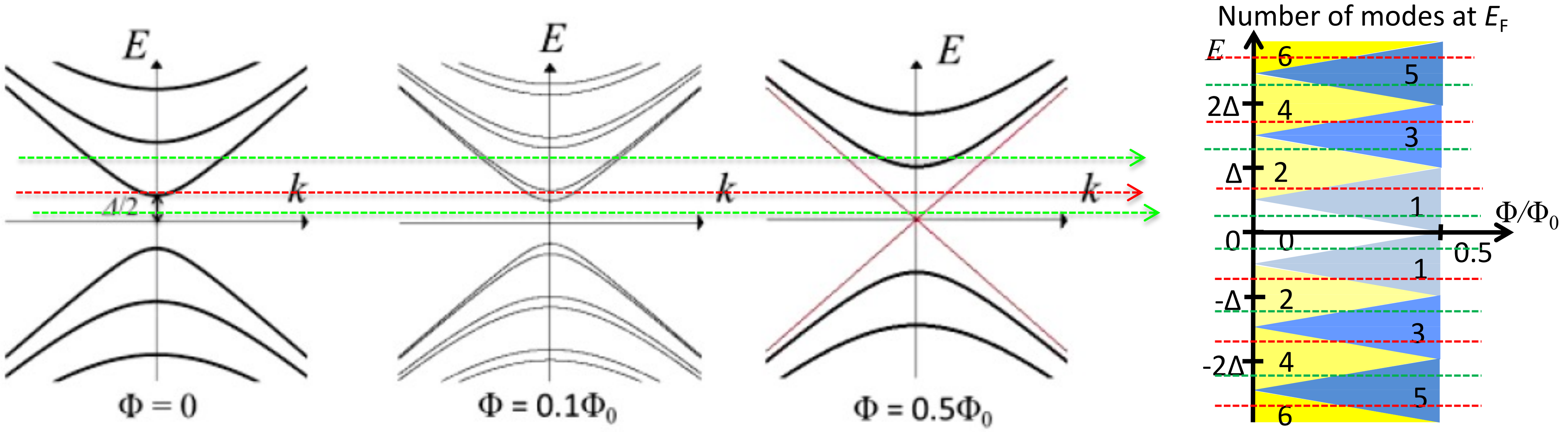
→ tuning of topologically protected, 1D helical mode!



- Bardarson *et al* PRL **105**, 156803 (2010).
- Zhang *et al* PRL **105**, 206601 (2010).
- Rosenberg *et al* PRB **82**, 041104(R) (2010)

Conductance of TI nanowire at Dirac point is expected to oscillate with a period of ϕ_0(h/e) and have a maximum at $\phi = \phi_0/2$ (a minimum at $\phi = 0$)

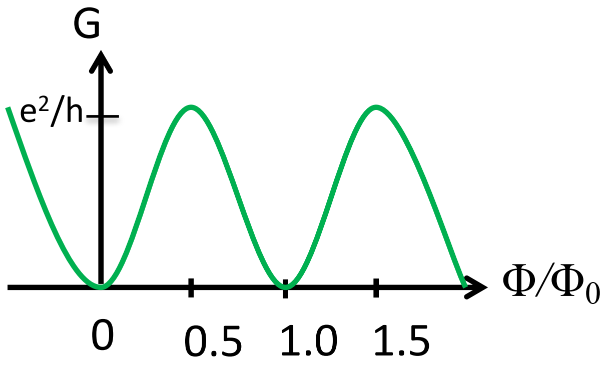
AB Oscillations: Signatures of the Gapless 1D-Topological Mode

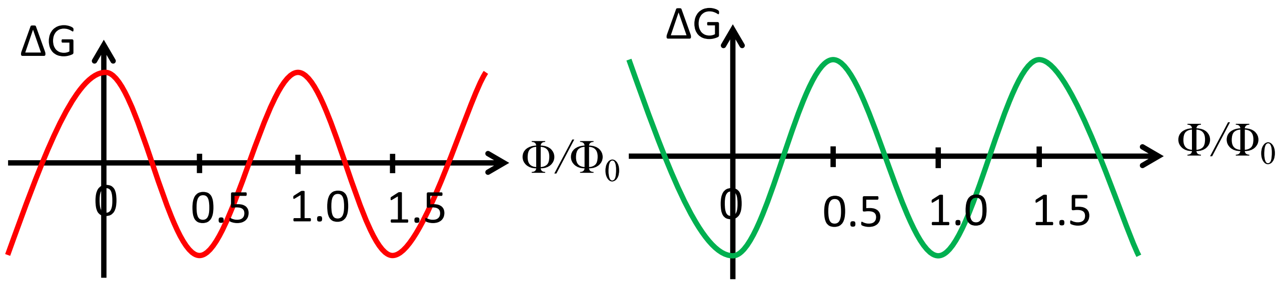


Topological mode gapped, other sub-bands doubly degenerate

Sub-band degeneracy split with field

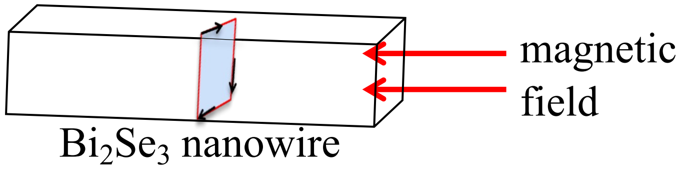
Topological mode present, other sub-bands doubly degenerate

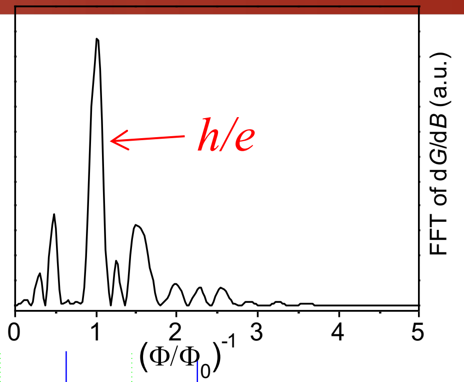




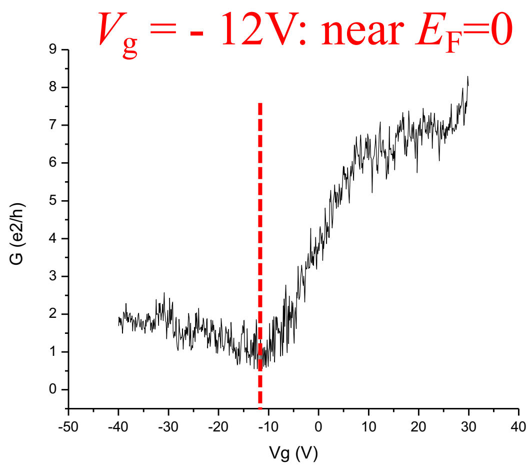
AB phase alternation with increasing E_F (V_{gate})

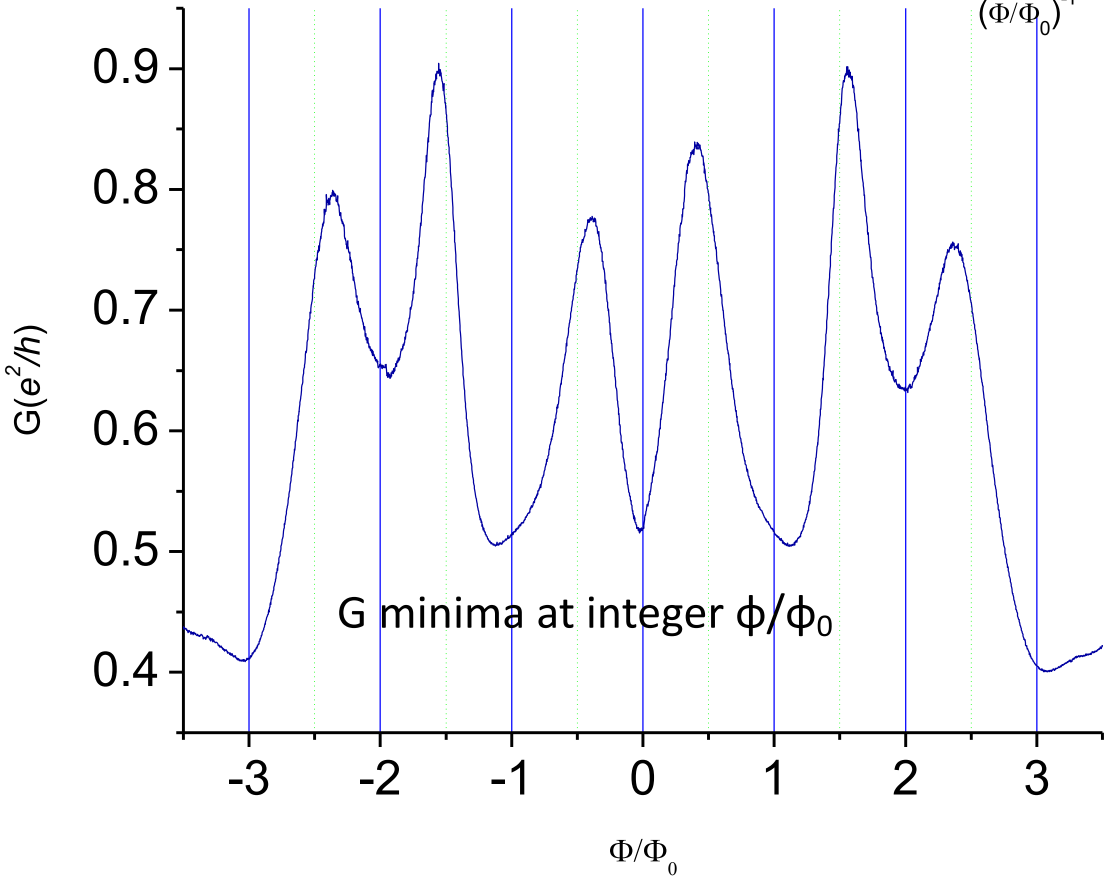
h/e oscillations at the Dirac point





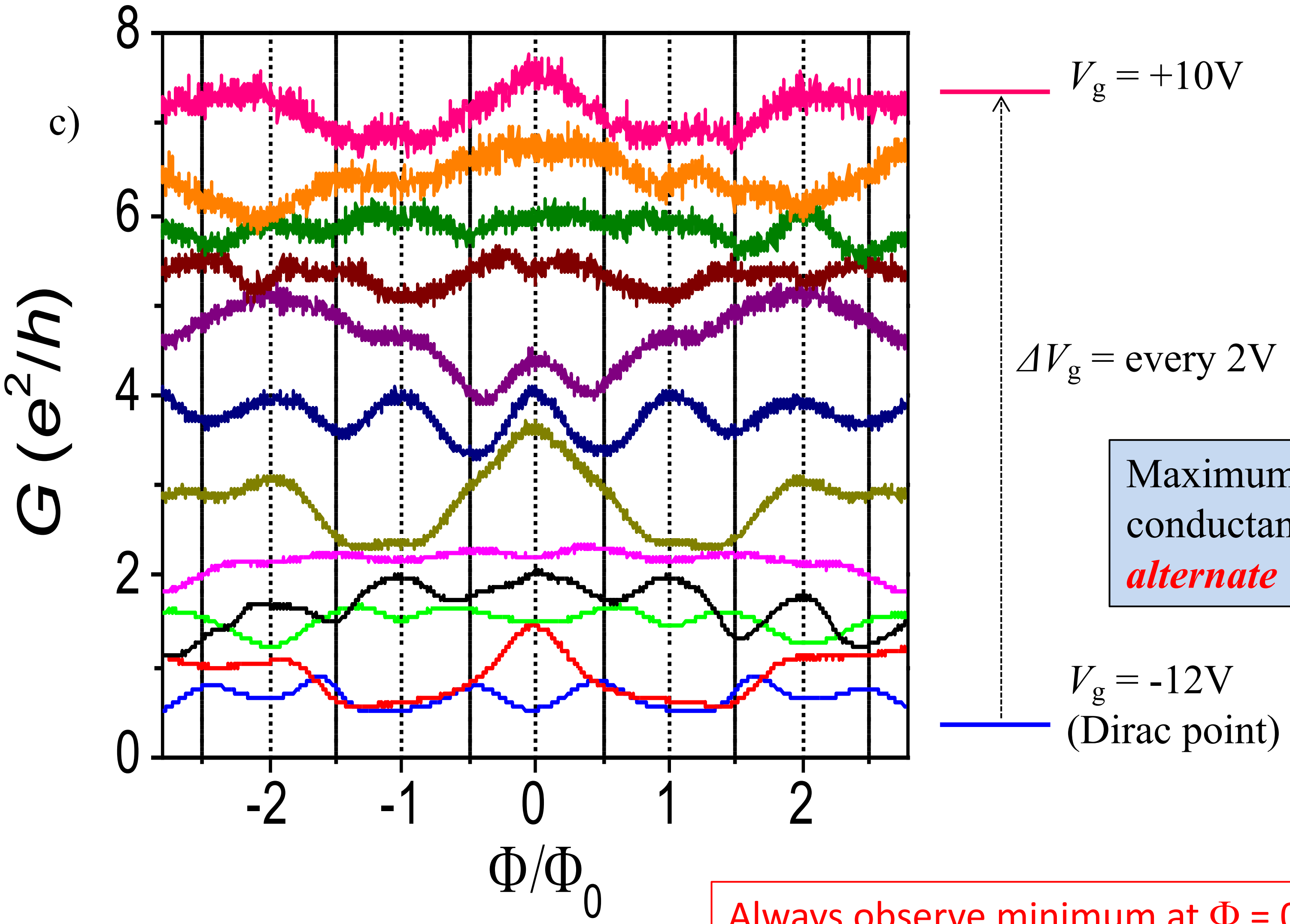
Dominant h/e (AB) rather than $h/2e$ (AAS)
→ low disorder

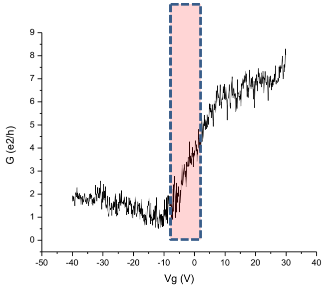




- Measurement of AB oscillation at the Dirac point
- G ~ e^2/h at $n = \phi/\phi_0 = 1/2$
- Consistent with theoretical expectation
- Clear signature of surface states and coherent transport

h/e oscillations: phase alternations with V_{Gate}

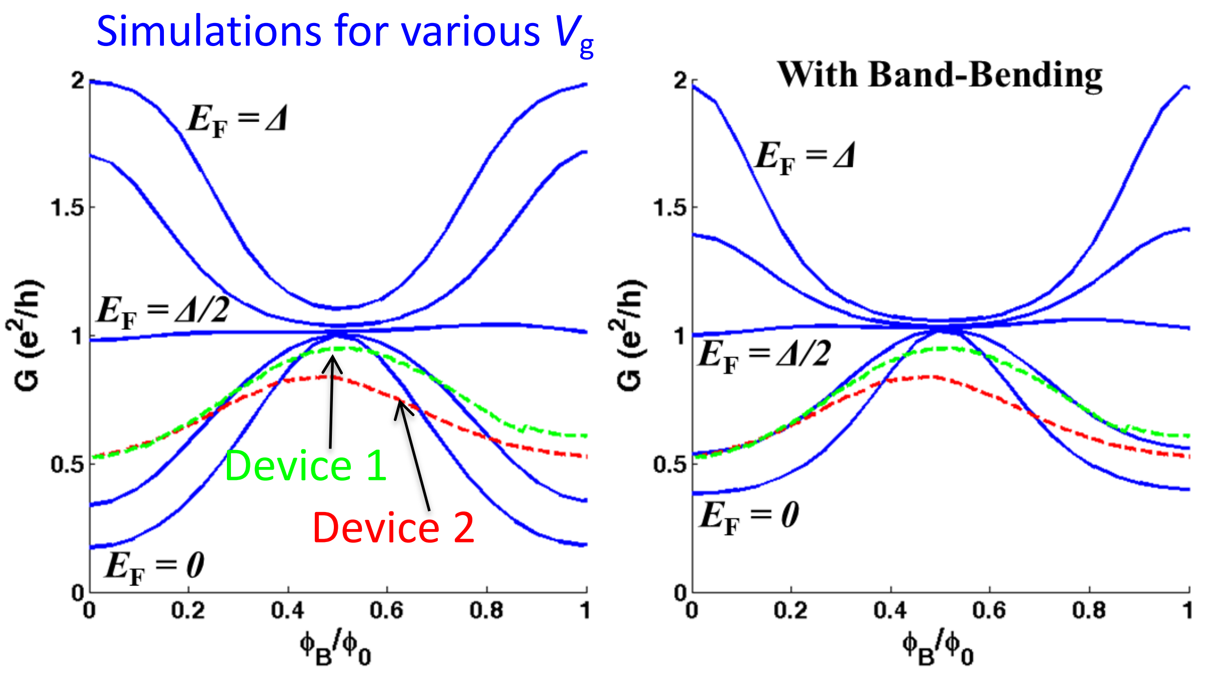




Maximum and minimum conductances at integer n ***alternate***

Always observe minimum at Φ = 0 near Dirac point

Simulations of AB oscillations (M.J. Gilbert, UIUC)



- Finite conductance at $\Phi = 0$ consistent with small shifts away from Dirac point, off-resonant conduction across contacts and slight band bending

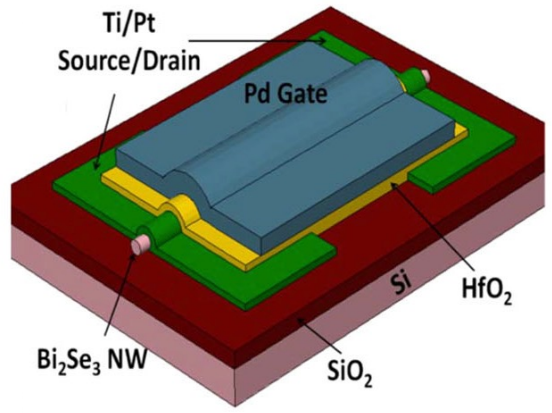
- Good fit between theory and experiment

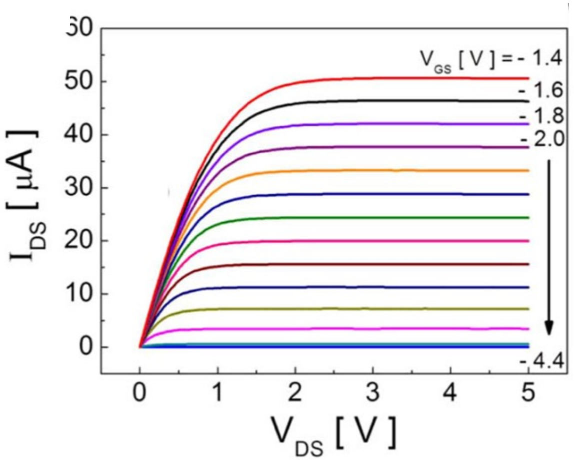
- AB data consistent with simulations for TI surface states
- Consistent with theoretical expectation: G ~ e^2/h at $n = \varphi/\varphi_0 = 1/2$ at Dirac point → Signature of 1D topological mode
- Evidence of 1D mode manipulation in TI nanowire
- Clear signature of surface states and coherent transport in 3D TI nanowire

Are surface states useful for devices? Maybe FETs or interconnects?

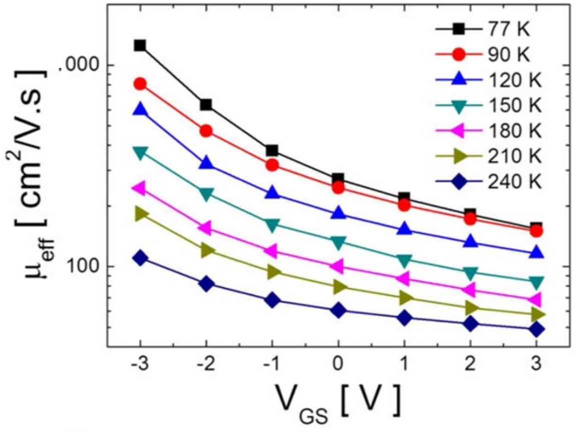
Example: TI FET

Benefits: No impurity scattering, dissipationless channel down to small scales





S. Cho et al, Nano Lett 11, 1925 (2011)

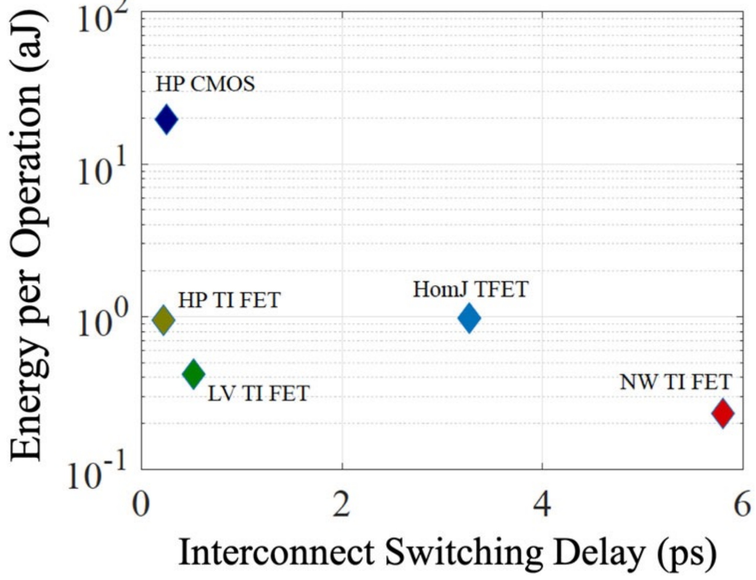


H. Zhu et al, Sci. Rep. 3, 1757 (2013)

Reality:

- Shows characteristic IV curves
- Low mobility due to low DOS

M. Gilbert, Comm. Phys. 4, 70 (2021)

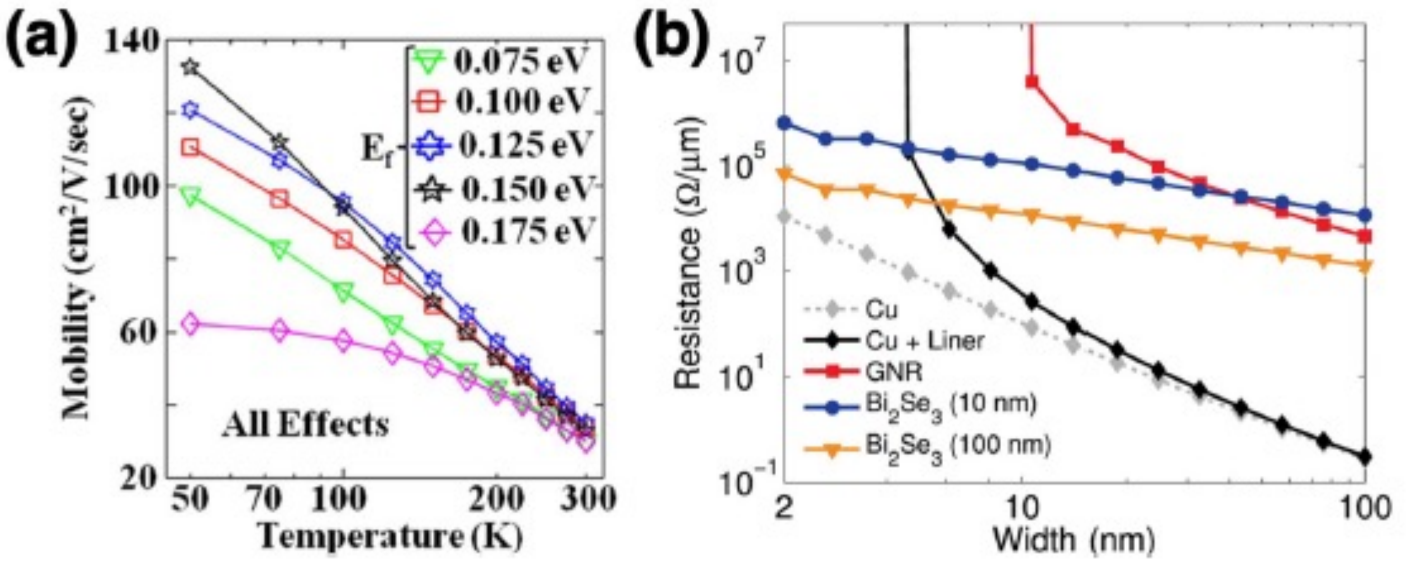


Potential:

- Theoretical benefits of high performance FETs
- Could use topological-to-nontopological switching
- Need greater control of surface state quality, Fermi energy location

TI Interconnects

Edge roughness and grain-boundary scattering make copper interconnects too resistive as they scale. Are TIs better?



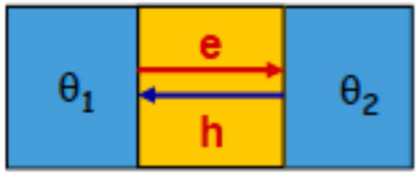
M. Gilbert, "Topological electronics," Comm. Phys. 4, 70 (2021)

Fig. 4 Topological interconnects. a Numerically calculated mobility of electrons in Bi_2Se_3 nanowire interconnects when considering non-ideal effects, such as roughness scattering and phonons, as a function of the simulation temperature for various Fermi levels[59]. Reprinted by permission from ref. [59], copyright 2014. **b** Theoretical resistance, normalized by the length, of different interconnect technologies compared to that of Bi_2Se_3 nanowire interconnects as calculated for two different mean-free paths of 10 nm (blue) and 100 nm (orange)[60].

Scattering & e-ph interactions decrease both bulk and surface mobility.
And density of states in surface states alone too small (compared to Cu)
However, may have use in ultra-narrow wires (< 10 nm)

But maybe surface states are "useful" in hybrid or quantum topological devices?

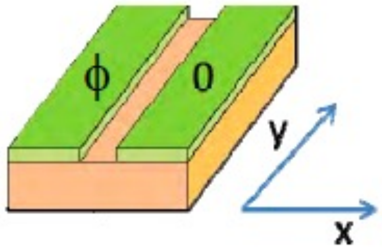
Topological Insulator-Superconducting Devices



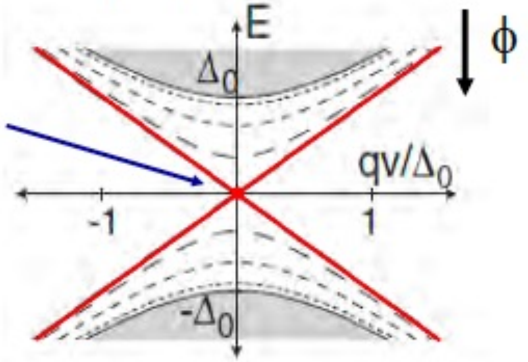
S-TI-S
topological insulator
barrier

Coherent electron-hole pair transport via topological surface state
"Andreev bound states" + "Majorana bound states" (zero-energy)

L. Fu and C. Kane, Phys. Rev. Lett. 100, 096407 (2008)



Zero-energy Majorana Bound states for $\phi = \pi$



ABS energy levels $$E_{\pm}(q,\phi) = \pm\sqrt{\upsilon^2 q^2 + \Delta^2 \cos^2(\phi/2)}$$

$$E_{\pm}(\phi) = \pm\Delta\cos(\phi/2)$$ for q=0

Supercurrent contribution $$I_{\pm}(\varphi) = \frac{2e}{\hbar}\frac{\partial E_{\pm}}{\partial\varphi} = \mp\left(\frac{e\Delta_0}{2\hbar}\right)\frac{\sin\varphi}{\sqrt{\frac{v^2q^2}{\Delta_0^2} + \cos^2\left(\frac{\varphi}{2}\right)}}$$

Low-energy ABS: $I \propto \cancel{\sin}\,\phi$ for q small, high transparency *(skewed CPR)*

Majorana states: $I \propto \sin\frac{\phi}{2}$ for q=0 *(4π-periodic CPR)*

Topological Insulator-Superconducting Devices

Lateral S-TI-S junctions for topological quantum computing?

Not the favorite system of most because of the complexity:

- 2D width (multiple channels)
- Multiple surfaces (top, edges, bottom)
- Conducting bulk states and trivial surface states in the TI

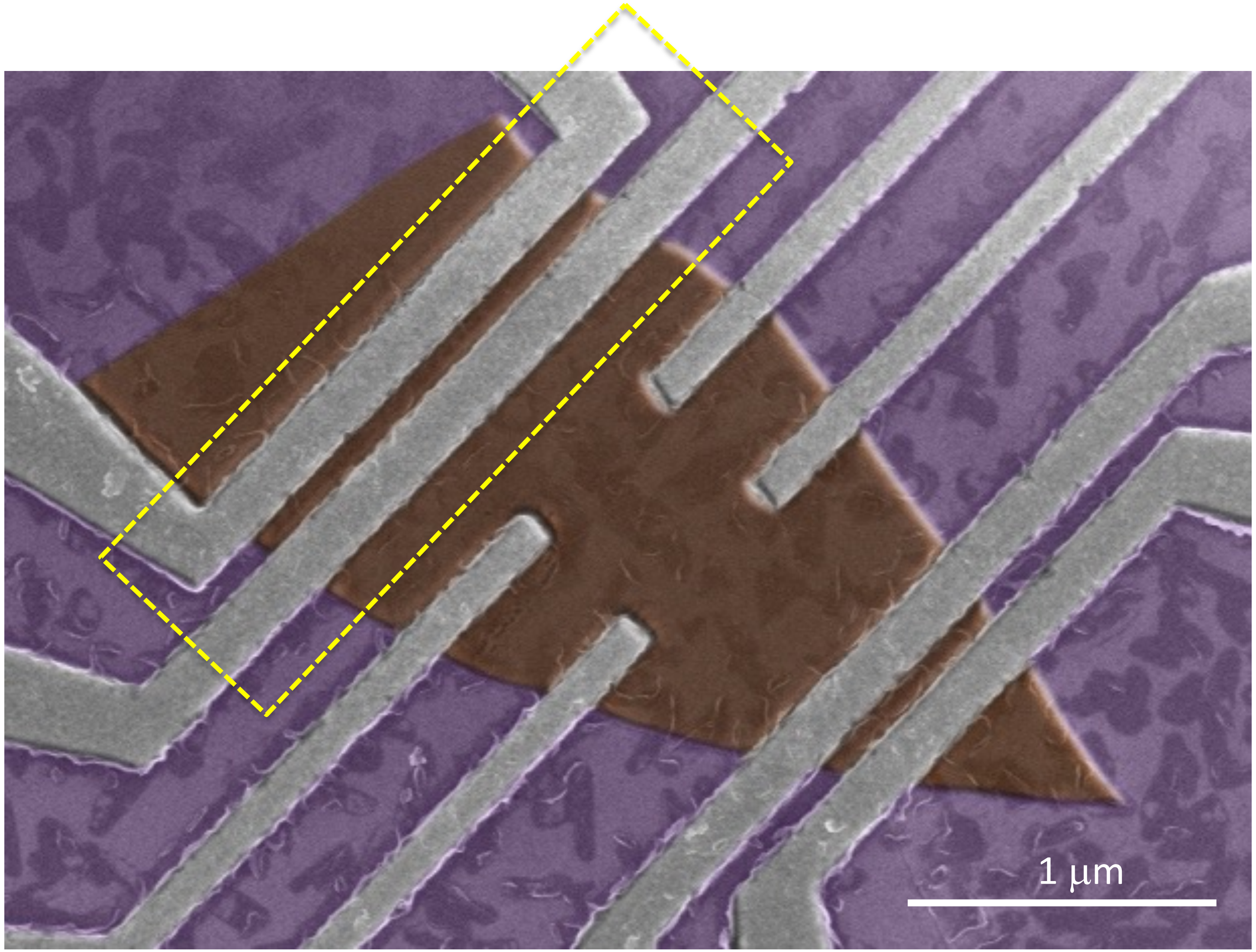
Advantages:

- Supports topological excitations without a strong magnetic field.
- Allows access to barrier for probes and imaging
- Expandable into networks
- Enables multiple modes of operation to control Majorana fermions by phase, current, or voltage
- Schemes proposed to braid and perform logical operations.

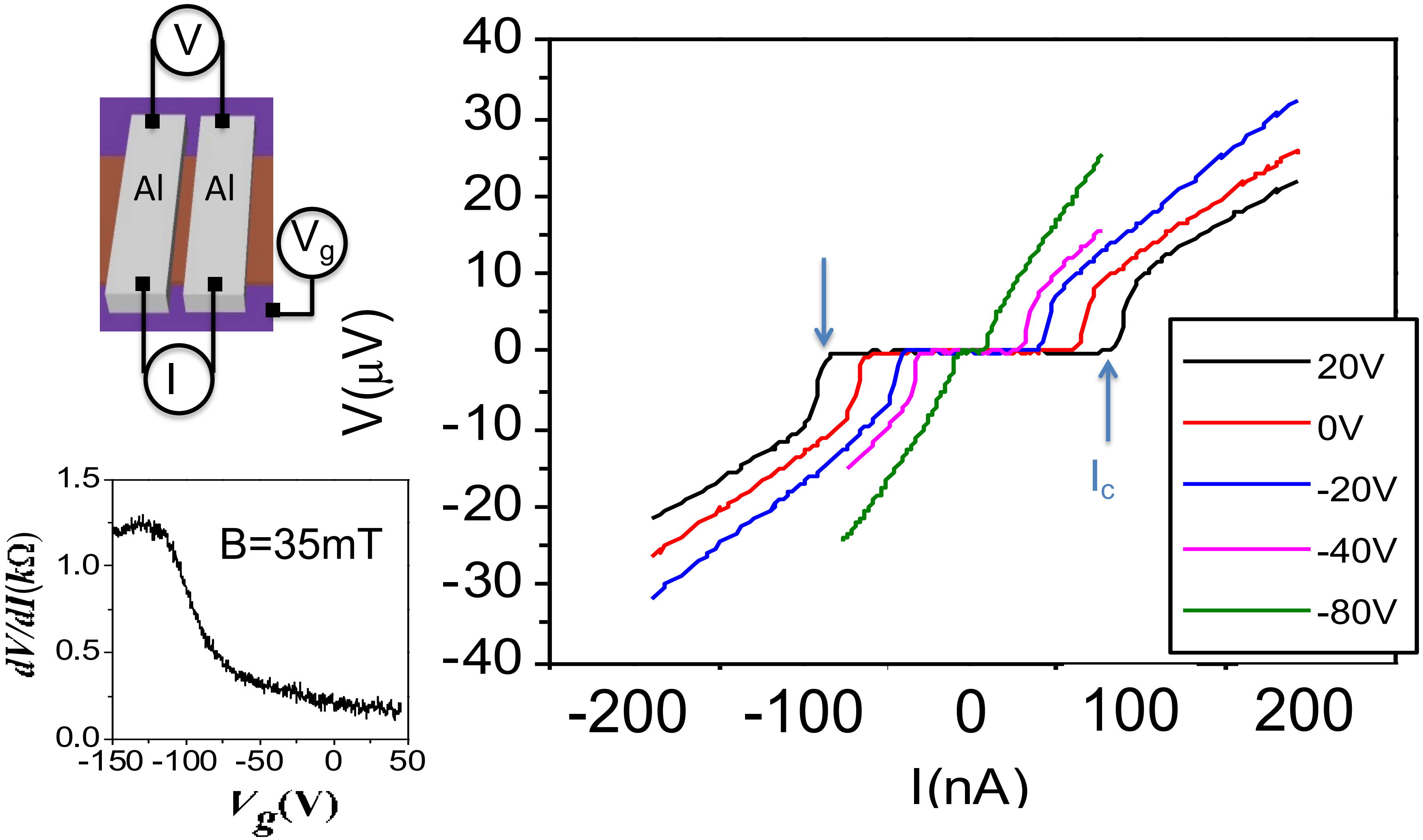
Trade-off stability for functionalization !

c.f. D. Van Harlingen, Quantum Science Summer School Lecture

Probing surface states in a S-TI-S junction

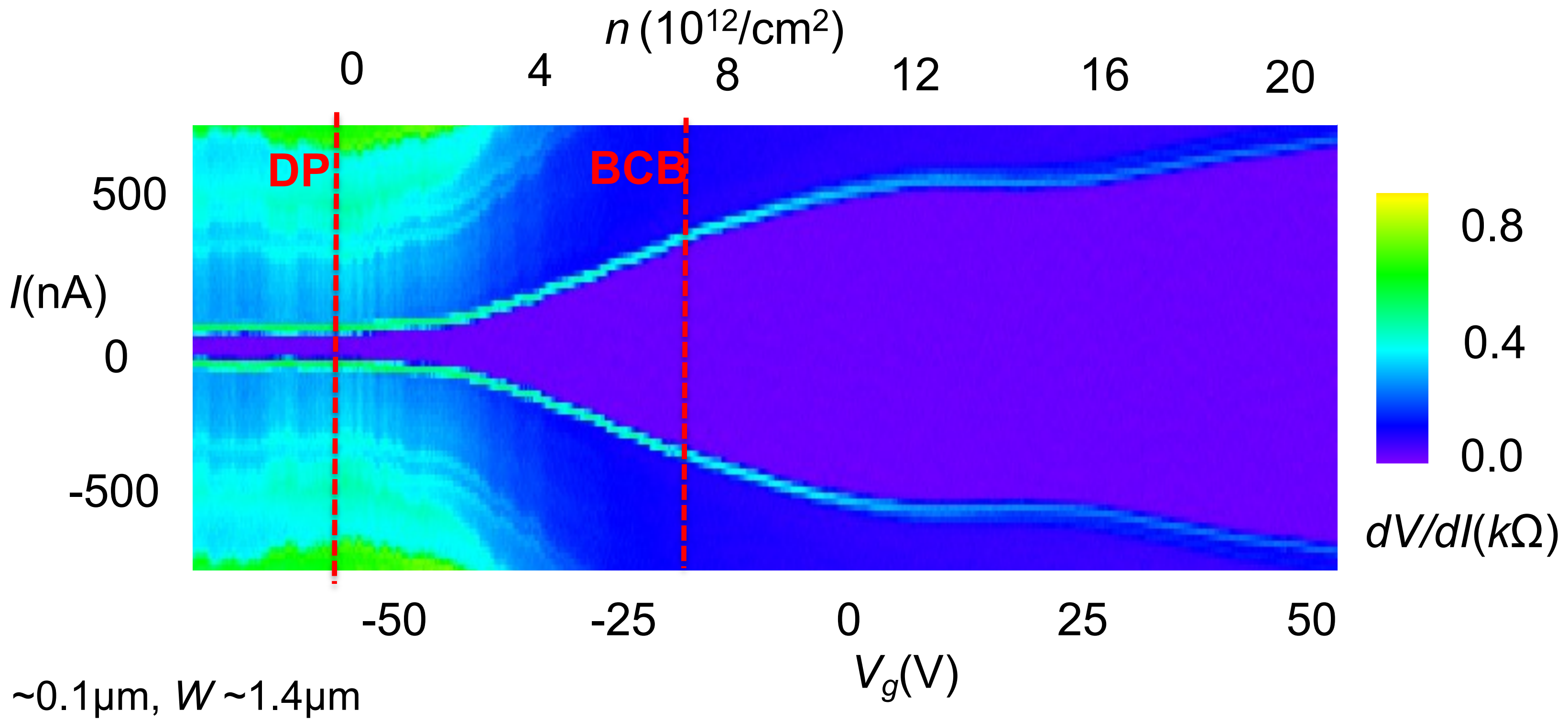


Gate-dependent supercurrent



I_c decreases as normal resistance in the junction increases

Supercurrents in the topological regime



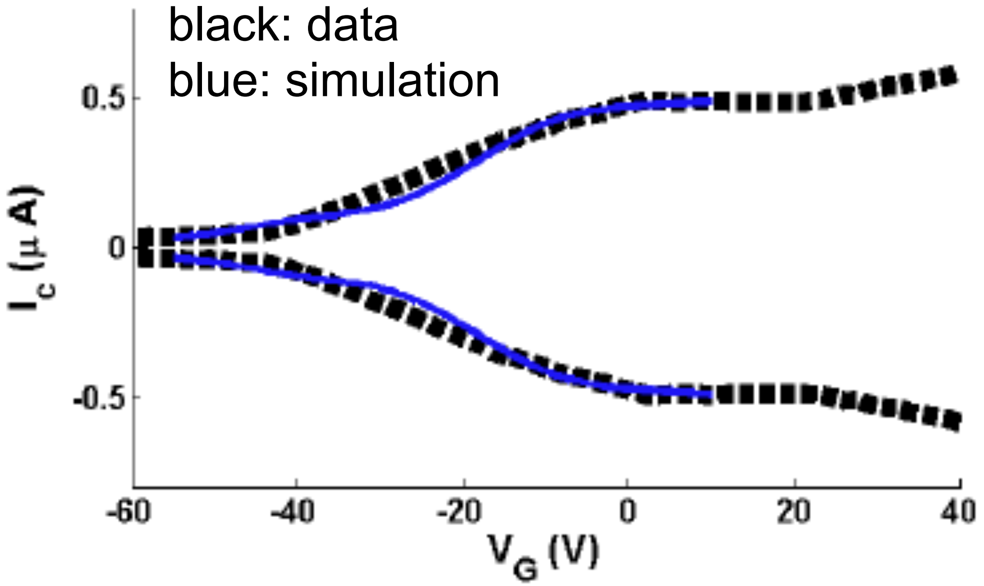
- L ~0.1μm, W ~1.4μm
- Purple area: supercurrents, boundary ~ I_c

- No change in I_c when pass into pure topological regime (past bottom of BCB)
- Finite I_c at Dirac point: residual densities in electron-hole puddles due to charged impurity potential.
- Critical current does not increase in hole region (V_g < -55V): asymmetric contact resistances or lack of clear surface states in valence band

Quantum Transport Theory: surface states carry supercurrent

Compare data to transport simulations considering superconducting leads and TI Hamiltonian

Supercurrents and density of states (DOS) profiles calculated in the non-equilibrium Green function formalism
(see Cho et al, Nat. Comm. 2013)



I_c closely follows surface DOS

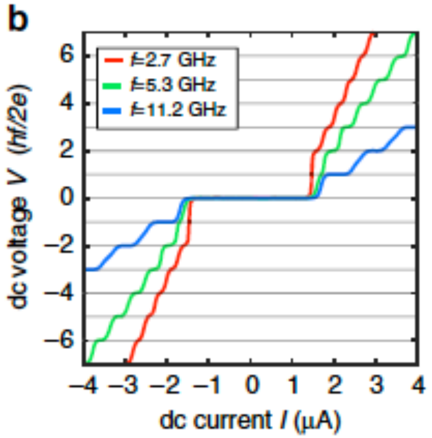
Adding surface disorder rapidly degrades supercurrent

→ surface states carry supercurrent!

TI Majorana Modes?

4π-periodic Josephson supercurrent in HgTe-based topological Josephson junctions

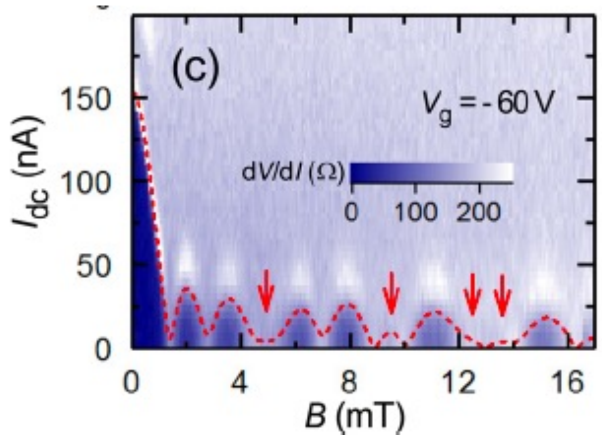
J. Wiedenmann[1,*], E. Bocquillon[1,*], R.S. Deacon[2,3,*], S. Hartinger[1], O. Herrmann[1], T.M. Klapwijk[4,5], L. Maier[1], C. Ames[1], C. Brüne[1], C. Gould[1], A. Oiwa[6], K. Ishibashi[2,3], S. Tarucha[3,7], H. Buhmann[1] & L.W. Molenkamp[1]



Evidence of Majoranas in 3D TIs not clear.
- No Majorana manipulation
- Disorder can cause anomalous junction behavior

Anomalous Fraunhofer Patterns in Gated Josephson Junctions Based on the Bulk-Insulating Topological Insulator $BiSbTeSe_2$

Subhamoy Ghatak,[†] Oliver Breunig,[†] Fan Yang,[*,†] Zhiwei Wang, Alexey A. Taskin, and Yoichi Ando[*]

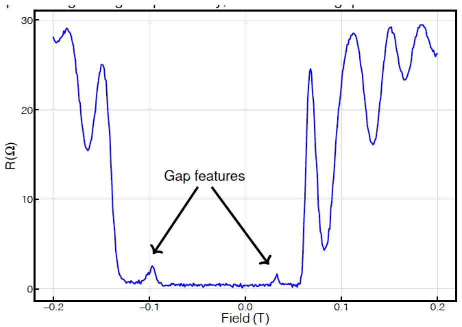


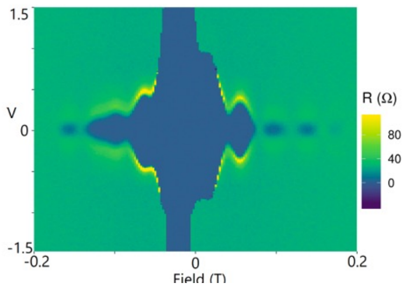
IOP Publishing
Journal of Physics: Condensed Matter
J. Phys.: Condens. Matter 33 (2021) 425601 (7pp)
https://doi.org/10.1088/1361-648X/ac15d7

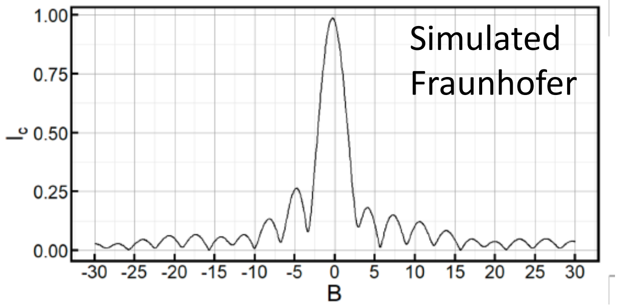
Asymmetric Fraunhofer spectra in a topological insulator-based Josephson junction

Alexander Beach, Dalmau Reig-i-Plessis, Gregory MacDougall and Nadya Mason*

Small, spatially varying disorder (like a step in the material) can add phases, induce anomalous Fraunhofer patterns and transport features







TI Majorana Modes?

New ideas for finding MZMs in TIs in wires …

PHYSICAL REVIEW B **104**, 165405 (2021)

Majorana bound states in topological insulators without a vortex

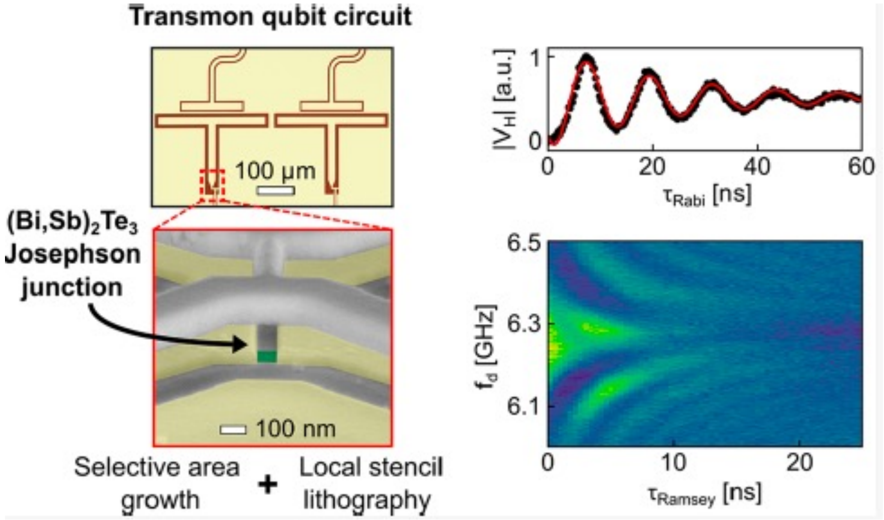
Henry F. Legg, Daniel Loss, and Jelena Klinovaja
Department of Physics, University of Basel, Klingelbergstrasse 82, CH-4056 Basel, Switzerland

(Received 24 March 2021; revised 17 September 2021; accepted 20 September 2021; published 4 October 2021)

We consider a three-dimensional topological insulator (TI) wire with a nonuniform chemical potential induced by gating across the cross section. This inhomogeneity in chemical potential lifts the degeneracy between two one-dimensional surface state subbands. A magnetic field applied along the wire, due to orbital effects, breaks time-reversal symmetry and lifts the Kramers degeneracy at zero momentum. If placed in proximity to an *s*-wave superconductor, the system can be brought into a topological phase at relatively weak magnetic fields. Majorana bound states (MBSs), localized at the ends of the TI wire, emerge and are present for an exceptionally large region of parameter space in realistic systems. Unlike in previous proposals, these MBSs occur without the requirement of a vortex in the superconducting pairing potential, which represents a significant simplification for experiments. Our results open a pathway to the realization of MBSs in present-day TI wire devices.

Integration of Topological Insulator Josephson Junctions in Superconducting Qubit Circuits

Tobias W. Schmitt, Malcolm R. Connolly,* Michael Schleenvoigt, Chenlu Liu, Oscar Kennedy, José M. Chávez-Garcia, Abdur R. Jalil, Benjamin Bennemann, Stefan Trellenkamp, Florian Lentz, Elmar Neumann, Tobias Lindström, Sebastian E. de Graaf, Erwin Berenschot, Niels Tas, Gregor Mussler, Karl D. Petersson, Detlev Grützmacher, and Peter Schüffelgen*



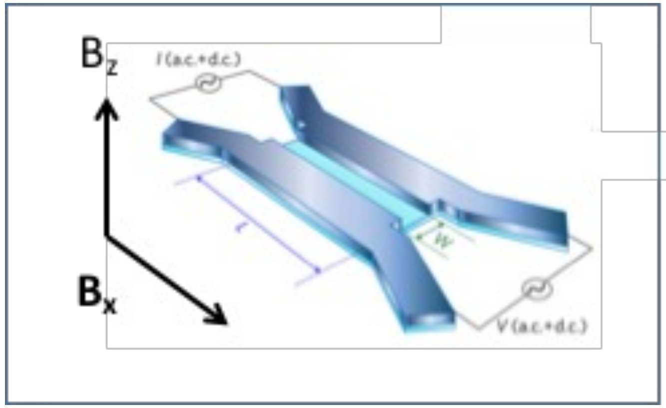
Emergent Physics in TI Devices

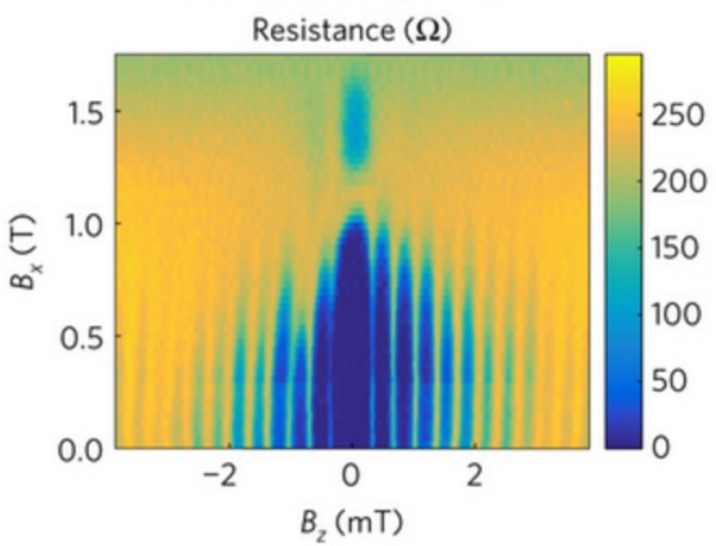
Can useful new electronic states be induced? For known or unknown applications?

Example: Fulde–Ferrell–Larkin–Ovchinnikov (FFLO) phase
- Finite momentum Cooper pairs with spatially non-uniform order parameter
- Very long-range superconducting pairing
- Could lead to superconducting diode effect (Yuan & Fu, PNAS, 2022)

Evidence of FFLO-like state (Cooper pairing with shifted momentum) in 2D TIs when in-plane field perpendicular to current is added

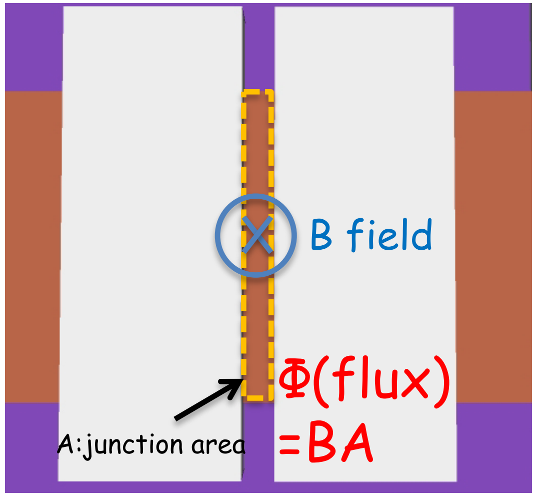
Hart et al., Nat. Phys. 13, 3877 (2017).

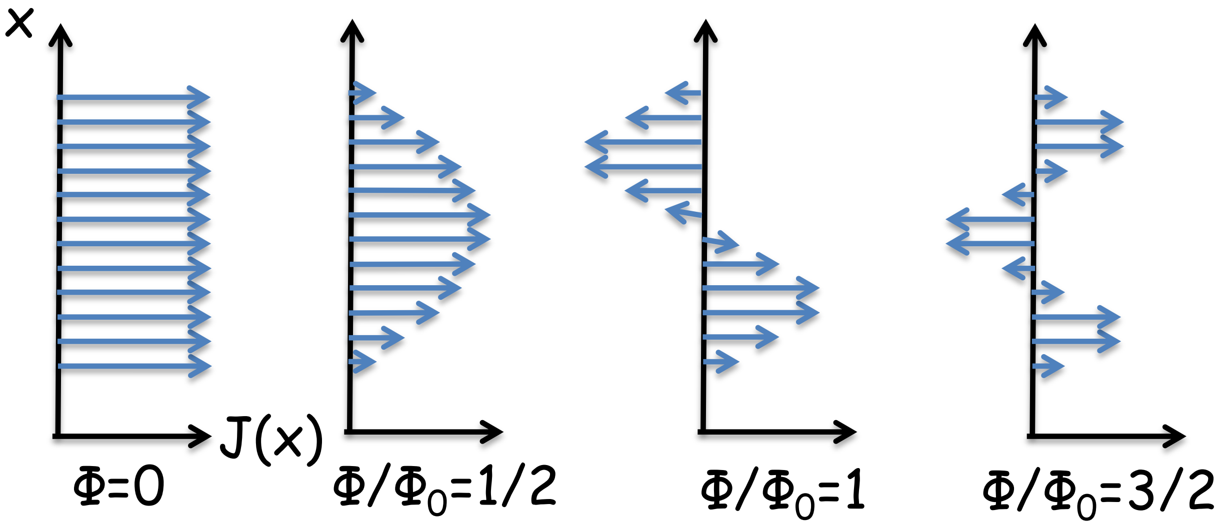




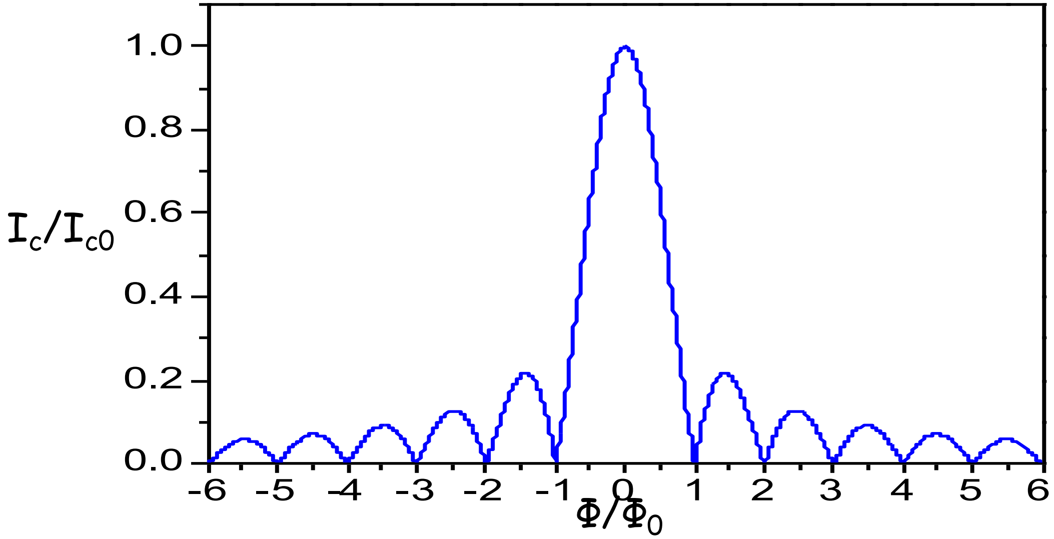
What about in a 3D topological insulator?

Finite momentum pairing studied via Fraunhofer Spectroscopy





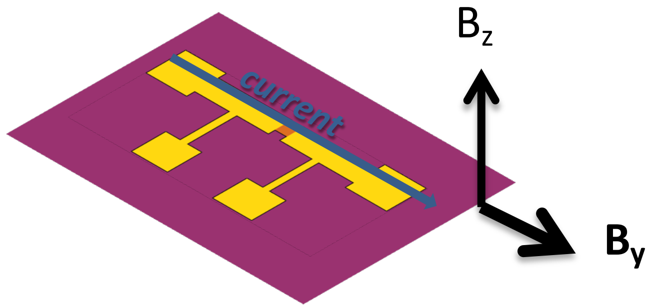
When magnetic field is applied through a uniform junction, the local Josephson current density oscillates sinusoidally with position due to single-valuedness of the phase



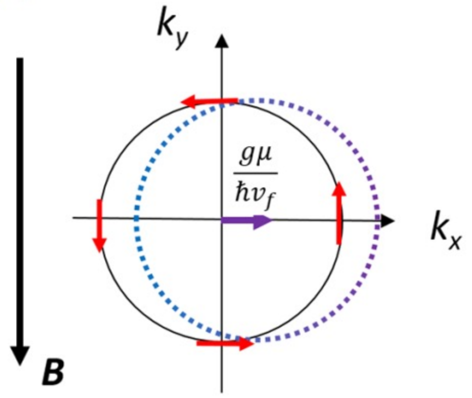
$$I_c = I_{c0} \left| \sin(\pi\Phi/\Phi_0)/(\pi\Phi/\Phi_0) \right|$$

Fraunhofer spectroscopy in 3D TIs

Adding an in-plane field (along current direction) shifts the momentum (and corresponding Dirac cone & Fermi surface)



$$H_{Dirac} = -\hbar v_F \left(k_x - \frac{g\mu B_y}{\hbar v_f}\right)\sigma_y + \hbar v_f k_y \sigma_x$$



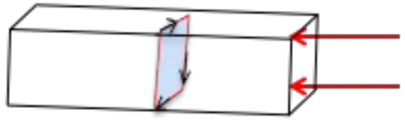
This gives center-of-mass momentum to Cooper pairs → $\Delta_{L,R} \approx \Delta_0 e^{i\frac{2g\mu B_y}{\hbar v_f}x}$

and adds a phase across the junction $\phi_1(x_1) - \phi_2(x_2) = \frac{2B_y(x_1 - x_2)g\mu}{\hbar v_f}$

"Zeeman term"

Fraunhofer spectroscopy in 3D TIs

An in-plane field also adds an Aharonov-Bohm term to the phase across the junction (this scales with thickness *t*):



$$\phi_1(x_1) - \phi_2(x_2) = \frac{\pi B_y(x_1 - x_2)t}{\Phi_0}$$

"AB term"

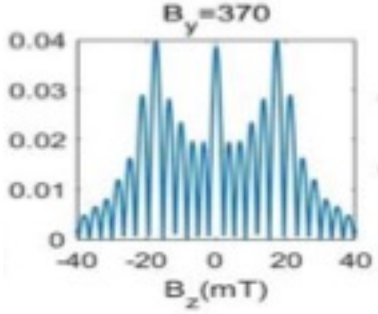
Using the total phase difference:

$$\phi_1(x_1) - \phi_2(x_2) = \frac{2\pi B_z d(x_1 + x_2)}{2\Phi_0} + \frac{2B_y(x_1 - x_2)g\mu}{\hbar v_f} + \frac{\pi B_y(x_1 - x_2)t}{\Phi_0}$$

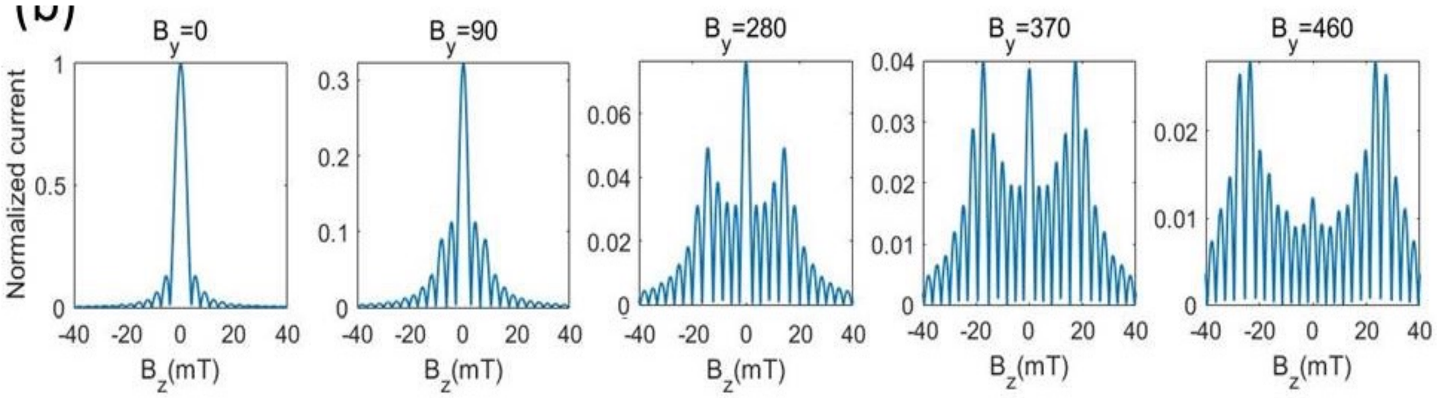
to model the transport current

$$I(\phi, B_y, B_z) = \int_{-\frac{W_1}{2}}^{\frac{W_1}{2}} \int_{-\frac{W_2}{2}}^{\frac{W_2}{2}} dx_1 dx_2 \frac{1}{d^2 + (x_1 - x_2)^2} \sin(\Delta\phi + \phi_1(x_1) - \phi_2(x_2))$$

gives simulated Fraunhofer patterns with in-plane field …



Simulated Fraunhofer spectroscopy (M. Gilbert, UIUC)

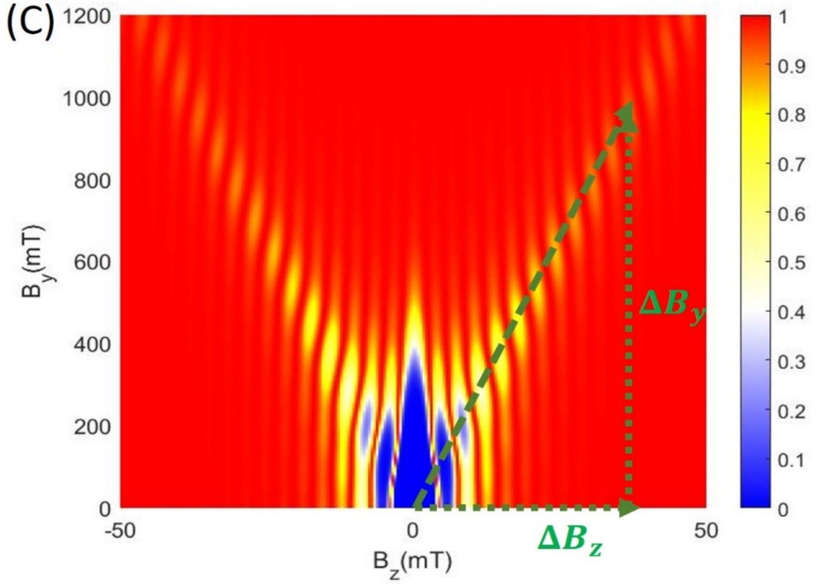


Simulations show spectral weight is shifted to larger B_z values as in-plane field B_y increased

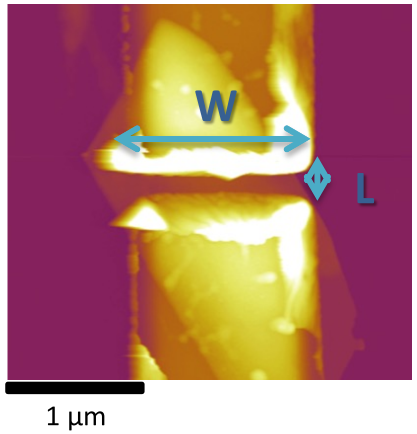
Leading to "trident" patterns in 2D Resistance plots of B_y vs B_z

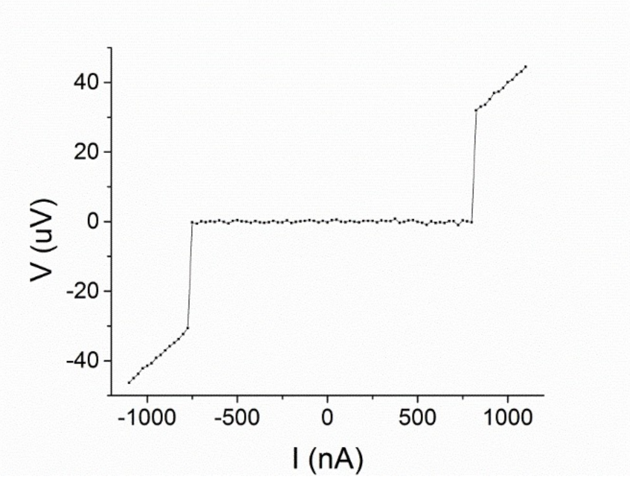
Where the slope depends on Zeeman and AB terms:

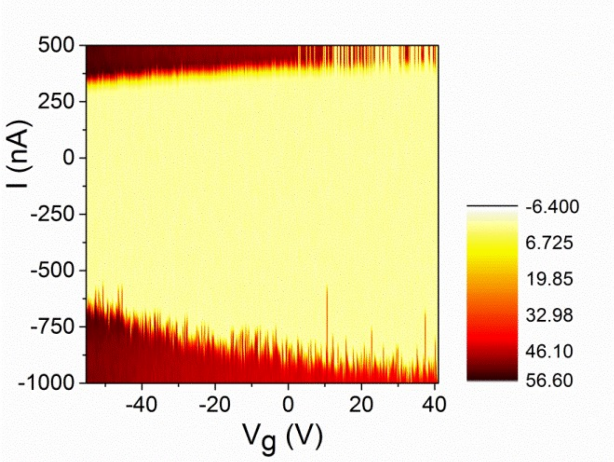
$$m = \frac{\Delta B_y}{\Delta B_z} = \frac{\pi d/\Phi_0}{\frac{2g\mu}{\hbar v_f} + \frac{\pi t}{\Phi_0}}$$



Fraunhofer device characteristics



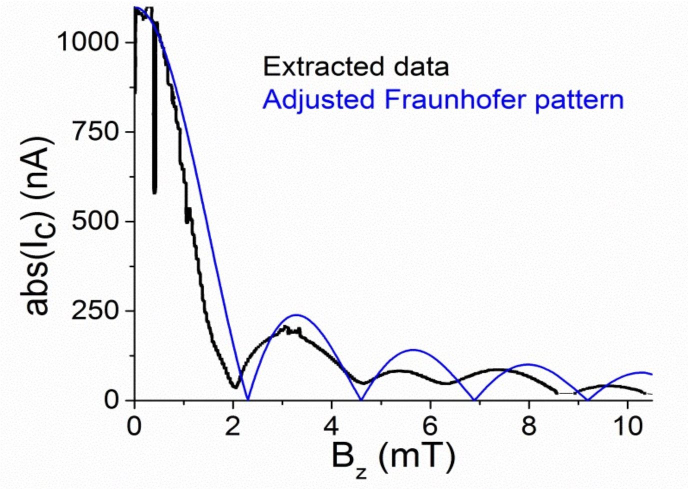




- Bi_2Se_3 flake, 120 nm x 1.5; thickness: 25 nm
- NbTiN/NbTi superconducting leads

Critical current ~ 1uA

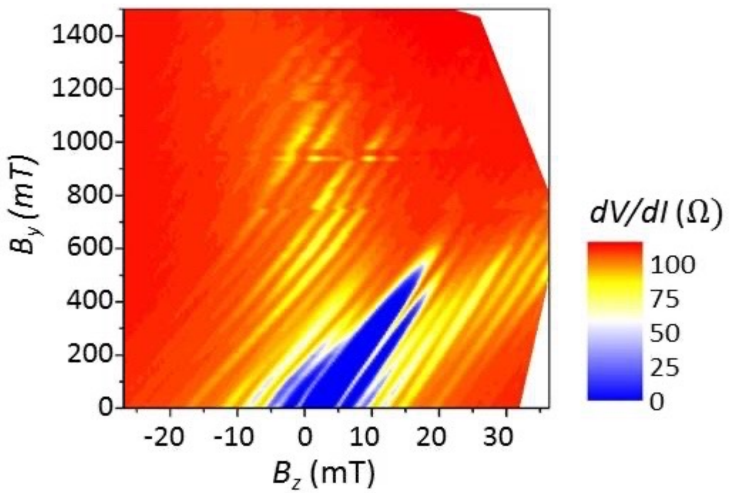
n-doped regime



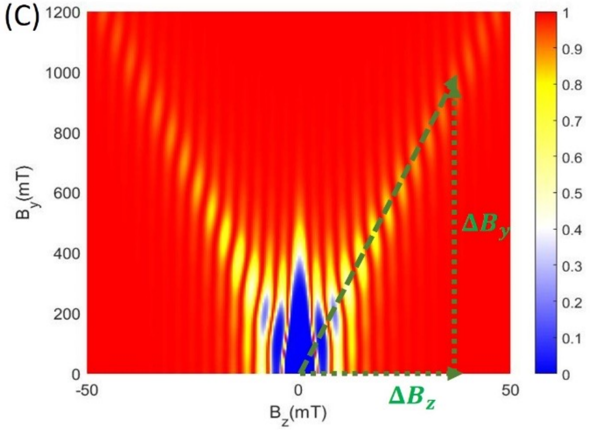
Standard Fraunhofer pattern without in-plane field

Fraunhofer spectroscopy

Data



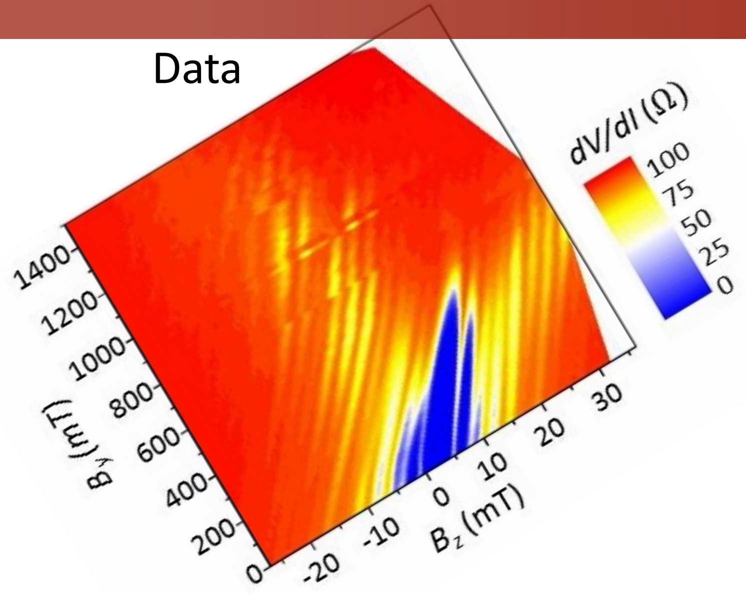
Simulation



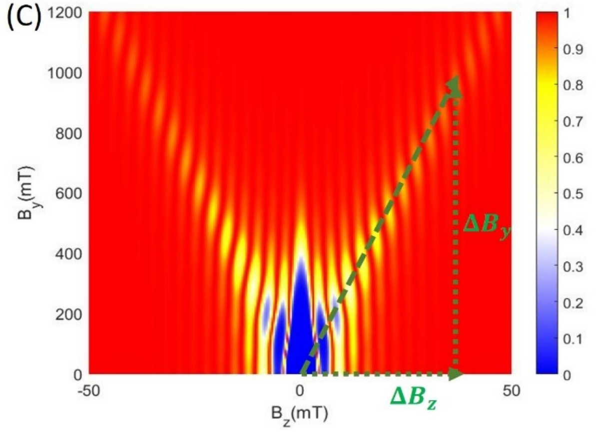
Data similar to simulation of finite momentum shifted Cooper pairs!

Fraunhofer spectroscopy

Data



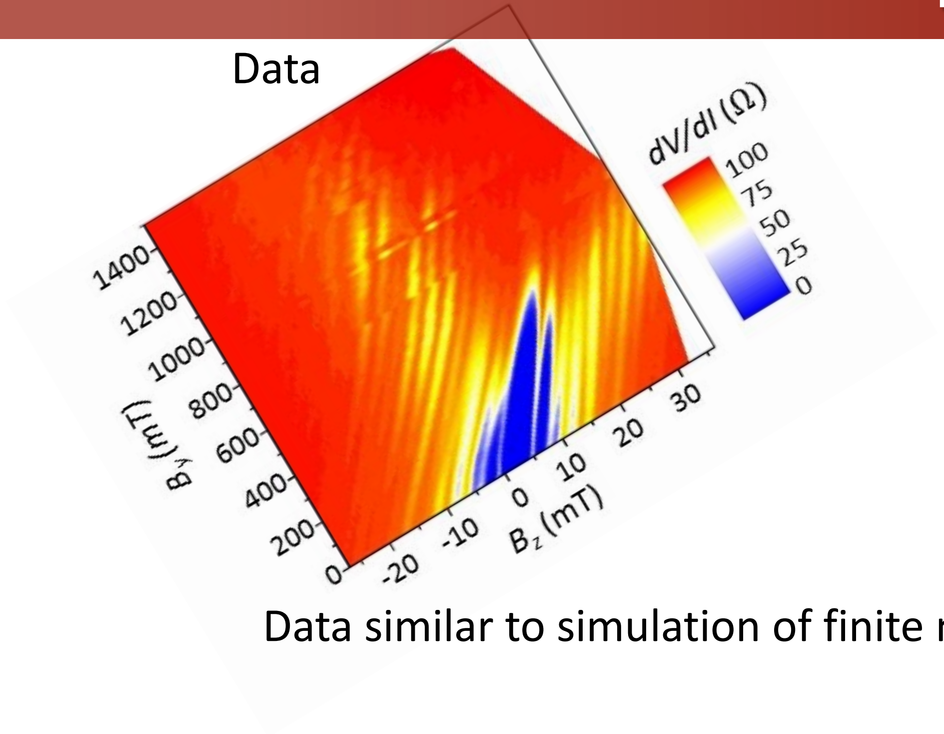
Simulation



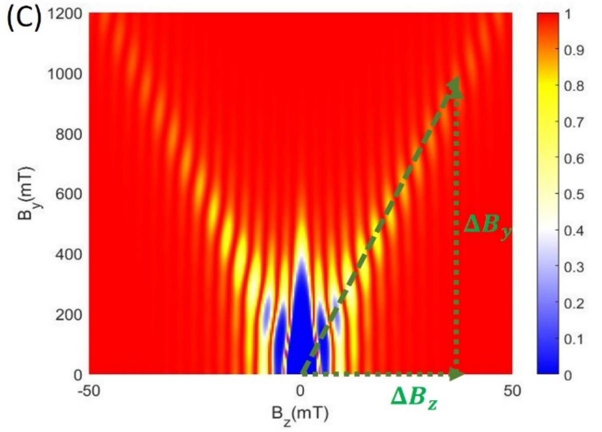
Data similar to simulation of finite momentum shifted Cooper pairs!

Fraunhofer spectroscopy

Data



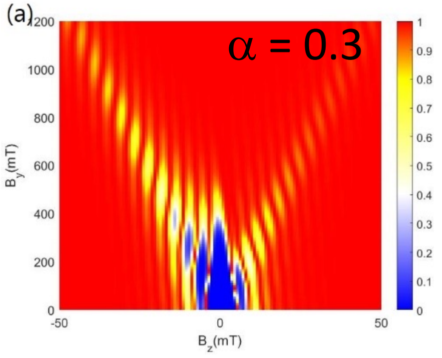
Simulation



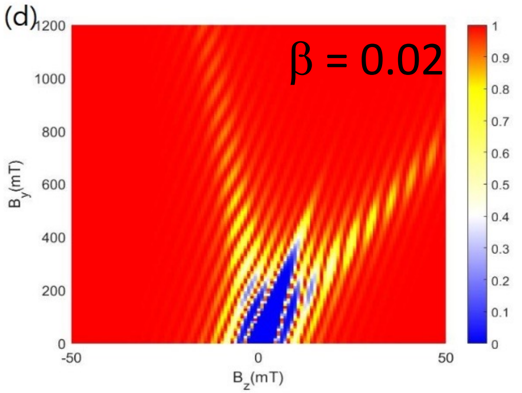
Data similar to simulation of finite momentum shifted Cooper pairs!

The "tilt" is due to the shape of the junction …

Asymmetry in lead widths, $\alpha = W_1/W_2$, gives asymmetry in B_z



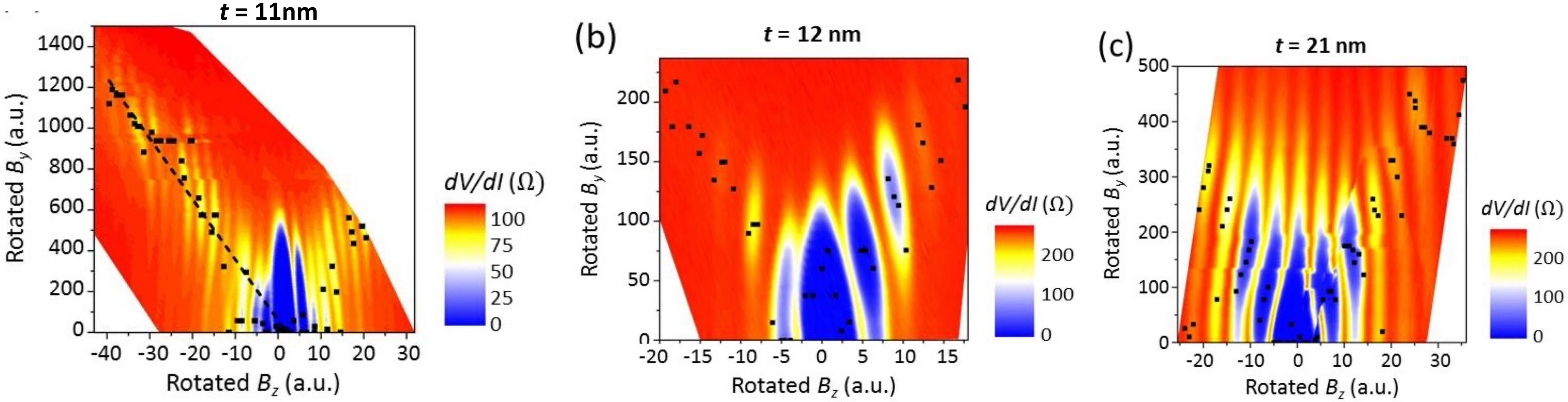
Flux focusing leads to tilt, i.e., $B_z \rightarrow B_z - \beta B_y$

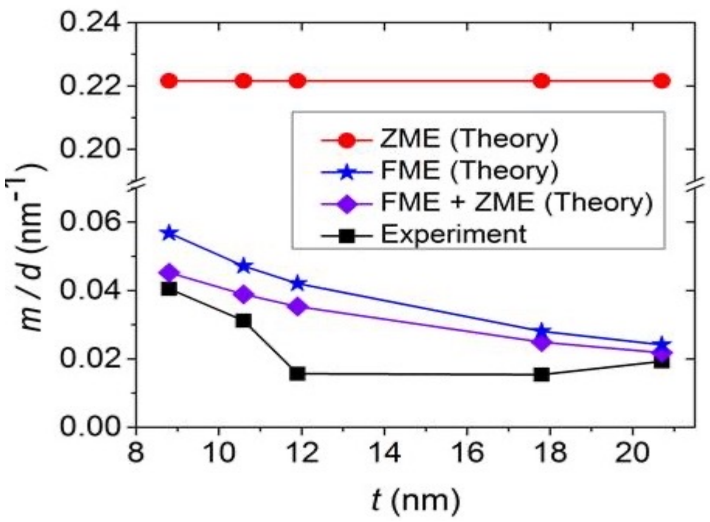


Fraunhofer spectroscopy

The slope versus thickness gives relative contribution of Zeeman and AB terms:

$$m = \frac{\Delta B_y}{\Delta B_z} = \frac{\pi d/\Phi_0}{\frac{2g\mu}{\hbar v_f} + \frac{\pi t}{\Phi_0}}$$





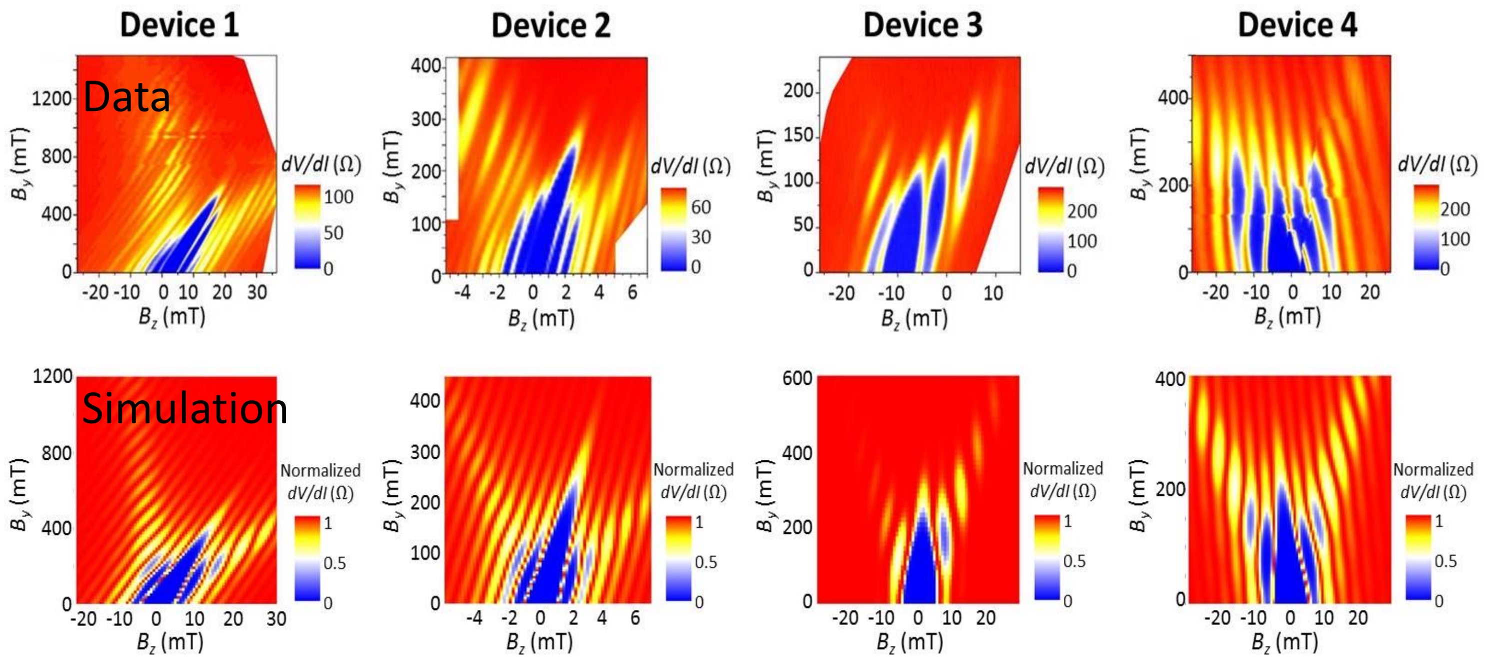
Plot of slope vs thickness shows suppressed values and trend with *t* not consistent with just AB → Zeeman contribution important

Fraunhofer spectroscopy

Device number	t (nm)	Average W (nm)	d (nm)	$\alpha = \frac{W_1}{W_2}$
1	9	920	110	1.07
2	11	1930	240	1.04
3	12	570	160	1.15
4	20	730	150	1.02
5	21	500	270	1.00

We observe finite momentum pairing!

We can simulate & understand Fraunhofer spectra for a wide variety of junctions!



Research Challenges in Topological Insulators

Topological insulators do not seem to have immediate application in conventional computing elements such as FETs & interconnects, or a Josephson devices

But … may be useful for special applications (e.g., low current devices) and also for studying new emergent states.

It’s really hard to displace Si/CMOS!

Some advances are needed, such as:

- Making materials having limited disorder
- Increasing topological gap energies
- Demonstrating topological properties at high temperatures
- Simple measurements that are sensitive to topology

There’s also a lot of potential in **Topological Spintronics**

Goldhaber-Gordon, Rechtsman, Mason, Armitage, Future Directions Workshop series: “Topological Sciences,” (2019)